PHYSICS

GRADE 12

STUDENT TEXTBOOK

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PHYSICS12

SEMESTER-ONE

PERIOD-1

P12CH1

TOPIC 1 Refraction and Dispersion of Light

In earlier grades you have studied about propagation of light and reflection of light. In this unit we will study about refraction of light.

1.1 BENDING OF LIGHT BETWEEN ADJACENT MEDIA

The change in direction of light when it passes from one medium to another is called refraction of light. In other words, the bending of light between adjacent media is called refraction of light.

ACTIVITY 1.1

Demonstrating refraction of light using air-water medium

Materials required

A coin, opaque bowl or cup and water

Procedure

- Place a large shallow bowl on a table and put a coin in it.
- Move away slowly from the bowl. Stop when the coin just disappears from your sight.
- Ask a friend to pour water gently into the bowl without disturbing the coin.
- Keep looking for the coin from your position.

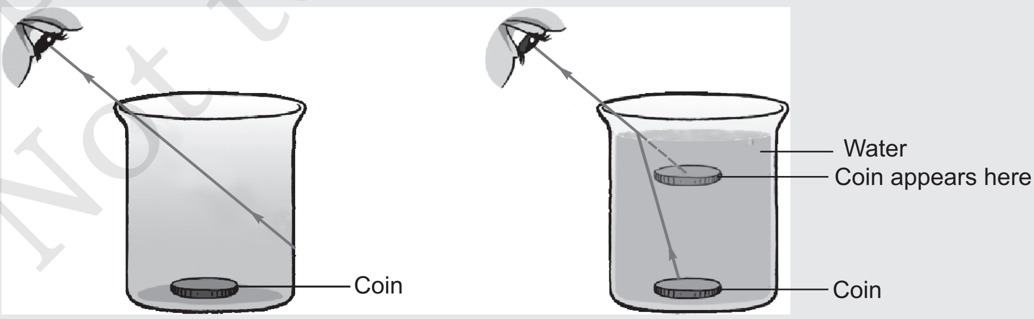


Fig. 1.1

Questions:

- Does the coin become visible again from your position?
- How could this happen?

The coin becomes visible again on pouring water into the bowl. This happens because the coin appears slightly raised above its actual position due to refraction of light.

Note: The diagrams used here show the container as transparent so that you can see the coin inside, whereas you will actually be using an opaque container.

ACTIVITY 1.2

Estabilishing experimentally the laws of refraction of light

- Fix a sheet of white paper on a drawing board using drawing pins.
- Place a rectangular glass slab over the sheet in the middle.
- Draw the outline of the slab with a pencil. Let us name the outline as ABCD.Take four identical pins.
- Fix two pins, say E and F vertically such that the line joining the pins is inclined to the edge AB.
- Look for the images of the pins E and F through the opposite edge. Fix two other pins, say G and H, such that these pins and the images of E and F lie on a straight line.
- Remove the pins and the slab.
- Join the positions of tip of the pins E and F and produce the line up to AE. Let EF meet AB at O. Similarly, join the positions of tip of the pins G and H and produce it up to the edge CD. Let HG meet CD at Q.
- Join O and O′. Also produce EF up to P, as shown by a dotted line in Fig. 1.2.

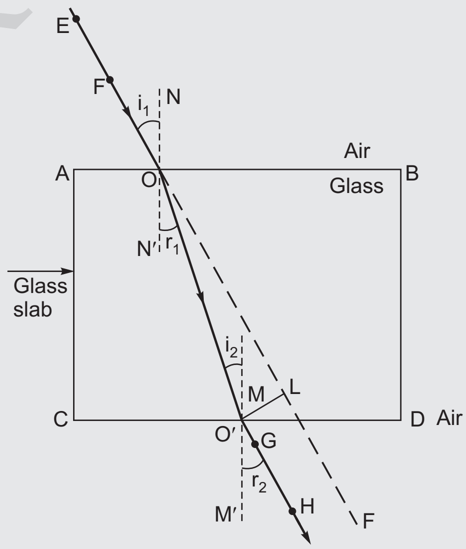


Fig. 1.2. Refraction of light through a rectangular glass slab

In this activity, you will note, the light ray has changed its direction at points *O* and *O′*. Note that both the points *O* and *O′* lie on surfaces separating two transparent media. Draw a perpendicular *NN′* to *AB* at *O* and another perpendicular *MM′* to *CD* at *O′*. The light ray at point *O* has entered from a rarer medium to a denser medium, that is, from air to glass. Note that the light ray has bent towards the normal. At *O′*, the light ray has entered from glass to air, that is, from a denser medium to a rarer medium. The light here has bent away from the normal. Compare the angle of incidence with the angle of refraction at both refracting surfaces *AB* and *CD*.

In Fig 1.2, *EO* is the incident ray, *OO′* is the refracted ray and *O′H* is the emergent ray. You may observe that the emergent ray is parallel to the direction of the incident ray. Why does it happen so? The extent of bending of the ray of light at the opposite parallel faces *AB* (air glass interface) and CD (glass air interface) of the rectangular glass slab is equal and opposite. This is why the ray emerges parallel to the incident ray. However, the light ray is shifted sideward slightly. What happens when a light ray is incident normally to the interface of two media? Try and find out.

Now you are familiar with the refraction of light. Refraction is due to change in the speed of light as it enters from one transparent medium to another. Experiments show that refraction of light occurs according to certain laws.

The following are the laws of refraction of light.

(*i*) *The incident ray, the refracted ray and the normal to the interface of two transparent media at the point of incidence, all lie in the same plane.*

(*ii*) *The ratio of sine of angle of incidence to the sine of angle of refraction is a constant for the light of a given color and for the given pair of media. This law is also known as Snell's law of refraction.*

If i is the angle of incidence and r is the angle of refraction, then,

$$\frac{\sin i}{\sin r} = \text{constant (denoted by } \mu \text{ or } n)$$

This constant value is called the refractive index of the second medium with respect to the first.

1.2 VERIFICATION OF SNELL'S LAW

ACTIVITY 1.3

Verifying Snell's law

Materials required

- A rectangular glass slab, white sheet of paper, drawing board and Pins.

Procedure

- Place a rectangular glass slab on a white sheet of paper fixed on a drawing board.
- Trace the boundary ABCD of the glass slab.
- Remove the glass slab and draw a normal N_1N_2 at O.
- Draw a straight line IO inclined at an angle say 30° with the normal. IO is the incident ray.
- Fix two pins P and Q on the incident ray IO.
- Place the glass slab within its boundary ABCD.

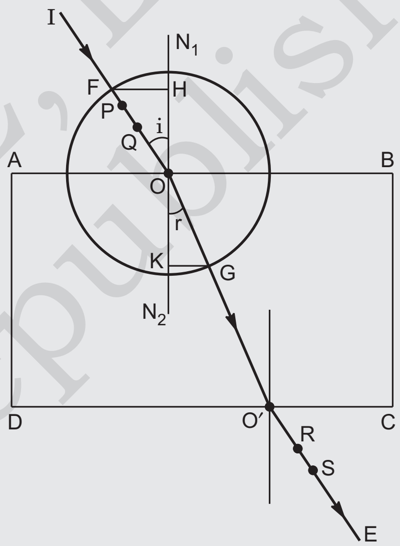


Fig. 1.3

- Looking from the other side of the glass slab fix two other pins R and S such that P, Q, R and S appear to lie on the same straight line.
- Remove the glass slab and the pins. Mark the pin points P, Q, R and S.
- Join the pins R and S and produce the line on both sides. The ray $O'E$ is the emergent ray.
- Join OO'. It is the refracted ray.
- With O as centre, draw a circle of a convenient radius 'r' in such a way that it cuts the incident and the refracted rays at F and G respectively.

- From F and G draw perpendiculars to the normal N_1N_2.
- DFHO and DGKO are right-angled triangles.

$$\therefore \qquad \sin i = \frac{FH}{OF}, \quad \sin r = \frac{GK}{OG}$$

$$\mu = \frac{\sin i}{\sin r} = \frac{FH}{OF} \times \frac{OG}{GK}$$

But $OG = OF = r$

So, $$\mu = \frac{FH}{OG} \times \frac{OG}{GK} \quad \text{or} \quad \mu = \frac{FH}{GK}$$

- Measure the length of FH and GK.
- Repeat the experiment for different values of angle of incidence.
- Record the result in tabular form.

Sl. No.	i	FH	GK	$\frac{FH}{GK}$
1.				
2.				
3.				
4.				
5.				

- Find the value of $\frac{FH}{GK}$ for different values of i.
- $\frac{FH}{GK}$ will be equal to a constant verifying Snell's law.

1.3 REFRACTIVE INDEX

ACTIVITY 1.4

Measuring refractive index of a glass block

Materials required

Glass block, light box (laser) styrofoam.

Procedure

1. Place glass block on a sheet of paper.
2. Trace the outline of the block on the sheet.

3. Remove the block.
4. 1/3 down the outline mark a tick on the page.
5. Using a protractor mark an angle such as 30° or 45°. This is your angle of incidence.
6. Complete lines to show the angles.
7. Set up the ray box so as a ray shines along this line.
8. On the other side of the block make a mark where the refracted ray leaves the block.

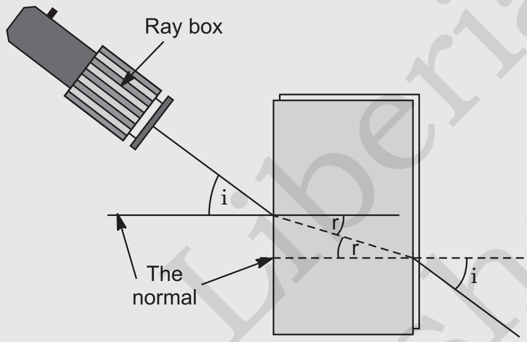


Fig. 1.4

9. Remove the block and complete the line from the point the light entered to where it left. This is the direction of the refracted ray.
10. Draw a normal and measure the angle r.
11. Having calculated r, calculate values of sin i and sin r using calculator.
12. Put the values in the following formula to get out the refractive index of glass block. Refractive index = $\sin i/\sin r = n$

1.4 REFRACTION OF LIGHT THROUGH LAYERS OF PARALLEL MEDIA

Consider a ray of light travelling from medium 1 into medium 2, as shown in Fig. 1.5. Let v_1 be the speed of light in medium 1 and v_2 be the speed of light in medium 2. The refractive index of medium 2 with respect to medium 1 is given by the ratio of the speed of light in medium 1 and the speed of light in medium 2. This is usually represented by the symbol n_{21}. This can be expressed in an equation form as,

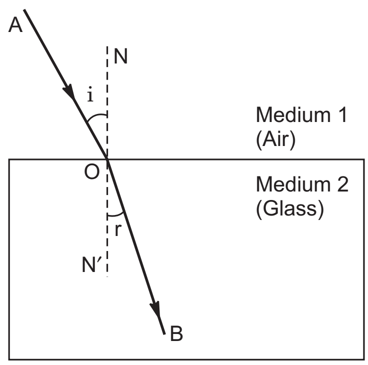


Fig. 1.5

$$n_{21} = \frac{\text{Speed of light in medium 1}}{\text{Speed of light in medium 2}} = \frac{v_1}{v_2}$$

By the same argument, the refractive index of medium 1 with respect to medium 2 is represented as n_{12}. It is given by

$$n_{12} = \frac{\text{Speed of light in medium 2}}{\text{Speed of light in medium 1}} = \frac{v_2}{v_1}$$

If medium 1 is vacuum or air, then the refractive index of medium 2 is considered with respect to vacuum. This is called the absolute refractive index of the medium. It is simply represented as n_2. If c is the speed of light in air and v is the speed of light in the medium, then, the refractive index of the medium n_m is given by

$$n_m = \frac{\text{Speed of light in air}}{\text{Speed of light in the medium}} = \frac{c}{v}$$

The absolute refractive index of a medium is simply called its refractive index. The refractive index of several media is given in Table 1.1. From the Table you can know that the refractive index of water, n_w = 1.33. This means that the ratio of the speed of light in air and the speed of light in water is equal to 1.33. Similarly, the refractive index of crown glass, n_g = 1.52.

Table 1.1 Absolute refractive index of some material media

Material medium	Refractive index
Air	1.0003
Ice	1.31
Water	1.33
Alcohol	1.36
Kerosene	1.44
Fused quartz	1.46
Turpentine oil	1.47
Benzene	1.50
Crown glass	1.52
Diamond	2.42

1.5 TOTAL INTERNAL REFLECTION OF LIGHT

ACTIVITY 1.5

Demonstrating total internal reflection of light

Materials required

A laser torch or pointer, a glass beaker with clear water and a glass test tube.

Procedure

1. Take a glass beaker with clear water in it. Stir the water a few times with a piece of soap, so that it becomes a little turbid. Take a laser pointer and shine its beam through the turbid water. You will find that the path of the beam inside water shines brightly.
2. Shine the beam from below the beaker such that it strikes at the upper water surface at the other end. Do you find that it undergoes partial reflection (which is seen as a spot and partial refraction [which comes out in the air and is seen as a spot on the roof; [Fig. 1.6 (*a*)]?

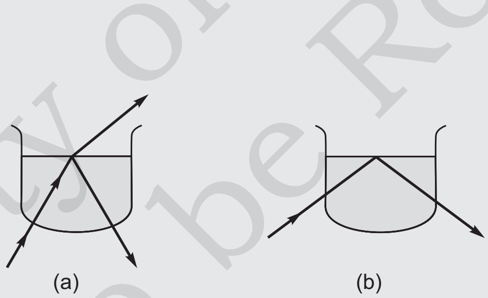


Fig. 1.6. Observing total internal reflection in water with a laser beam (refraction due to glass of beaker neglected being very thin).

3. Now direct the laser beam from one side of the beaker such that it strikes the upper surface of water more obliquely [Fig. 1.6 (*b*)]. Adjust the direction of laser beam until you find the angle for which the refraction above the water surface is totally absent and the beam is totally reflected back to water. This is total internal reflection at its simplest.
4. Pour this water in a long test tube and shine the laser light from top, as shown in Fig. 1.6 (*c*). Adjust the direction of the laser

beam such that it is totally internally reflected every time it strikes the walls of the tube. This is similar to what happens in optical fibres.

Caution: Take care not to look into the laser beam directly and not to point it at anybody's face.

1.5.1 Critical Angle

It is that angle of incidence for which a ray of light while moving from a denser to a rarer medium just grazes over the surface of separation of the two media (that is, angle of refraction = 90°).

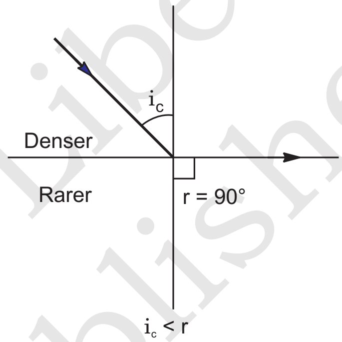


Fig. 1.7

1.5.2 Total Internal Reflection

If the angle of incidence of a ray of light traveling from a denser medium to rarer medium is greater than the critical angle for the two media, then the ray is reflected into denser medium and this phenomenon is described as **total internal reflection**.

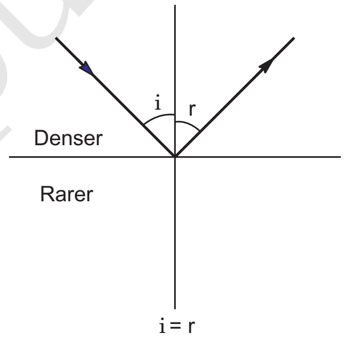


Fig. 1.8

Conditions to be satisfied for total internal reflection to take place

The conditions to be satisfied for total internal reflection to take place are:

- The ray of light must travel from a denser medium to a rarer medium.
- The angle of incidence must be greater than the critical angle for those two mediums.

Table 1.2 Critical angle of some transparent media with respect to air.

Substance medium	Refractive index	Critical angle
Water	1.33	48.75°
Crown glass	1.52	41.14°
Dense flint glass	1.62	37.31°
Diamond	2.42	24.41°

1.6 APPLICATIONS OF TOTAL INTERNAL REFLECTION

Total internal reflection has the following applications.

1.6.1 Optical Fibres

An optical fibre is a thin rod of high-quality glass. Very little light is absorbed by the glass. Light getting in at one end undergoes repeated total internal reflection, even when the fibre is bent, and emerges at the other end.

- Optical fibres are used in endoscopes that allow surgeons to see inside their patients.
- Optical fibres can also carry enormous amounts of information as pulses of light.

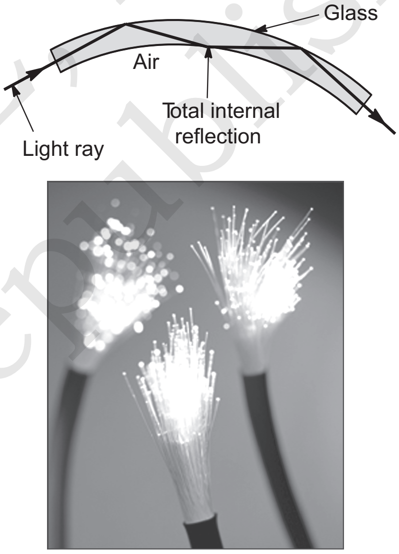


Fig. 1.9. Optical fibres

1.6.2 Diamond

The brilliance of diamond is due to total internal reflection. The critical angle for diamond-air interface (24.4°) is very small, therefore once light enters a diamond, it undergoes total internal reflection inside it. The faces of the diamond are so cut that a ray of light entering the diamond fall at angle greater than 24.4°. This results in multiple, total internal reflections

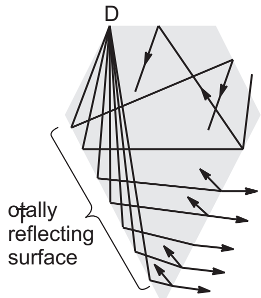


Fig. 1.10. Total internal reflection in diamond

at various angles and remains within the diamond. Hence diamond sparkles.

1.6.3 Prism

ACTIVITY 1.6

Illustrating total internal reflection of light using a right-angled prism.

A right angled glass prism can be used to change the direction of a light ray by 90 degrees or 180 degrees. Such prisms make use of total internal reflection [Fig. 1.11 (*a*) and (*b*)]. The light ray enters the prism along a normal and continues straight on until it hits the back face of the prism. Total internal reflection occurs here because light strikes the surface at 45 degrees which is greater than the critical angle of the glass prism (We see from Table 1.2 that this is true for both crown glass and dense flint glass). The light ray then emerges from the prism along a normal and so continues straight through the glass surface. This type of prism can be used in a periscope.

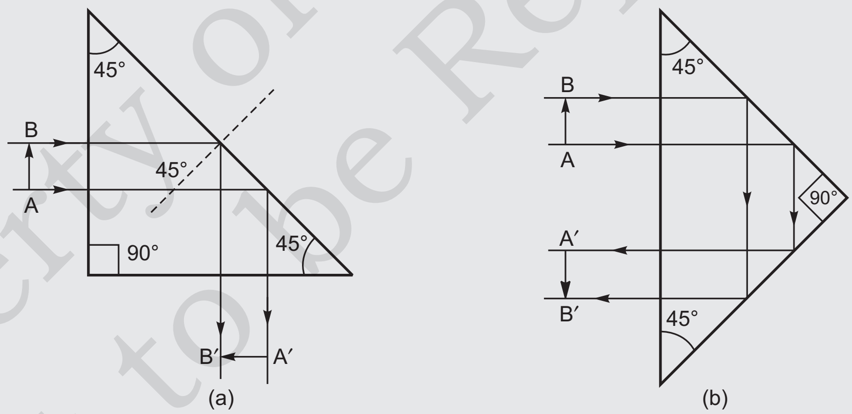


Fig. 1.11. Prisms designed to bend rays by 90° and 180° making use of total internal reflection.

1.6.4 Mirage

On hot summer days, the air near the ground becomes hotter than the air at higher levels. The refractive index of air increases with its density. Hotter air is less dense, and has smaller refractive index than the cooler air. As a result, light from a tall object such as a tree, passes through a medium whose refractive index decreases towards the ground. Thus, a ray of light from such an object successively bends away from the normal and undergoes total internal reflection, if the angle of incidence for the air near the ground exceeds the critical angle. This is shown in Fig. 1.12.

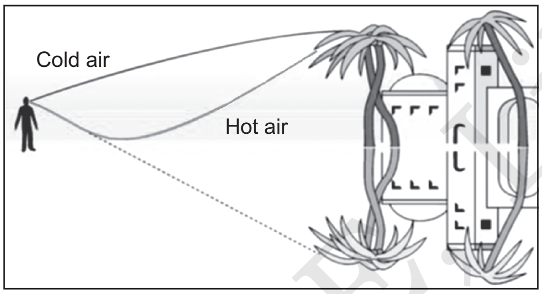


Fig. 1.12. Phenomenon of mirage

To a distant observer, the light appears to be coming from somewhere below the ground. The observer naturally assumes that light is being reflected from the ground, say, by a pool of water near the tall object. Such inverted images of distant tall objects cause an optical illusion to the observer. This phenomenon is called *mirage*.

1.7 REFRACTION OF LIGHT THROUGH A THIN LENS

Usually we see a palmist using a magnifying glass to see the lines on the palm of a person. This magnifying glass is called a lens.

A piece of a transparent medium bounded by two (at least one) spherical surfaces, is called spherical lens.

Working of a lens is based on the refraction of rays of light when they pass through the lens. Lenses do not have uniform thickness.

There are two types of spherical lense:

(*i*) **Convex or Converging Lenses:** It is thick in the middle and thin at the edges.

All types of convex lenses refract a parallel beam of light inwardly.

As these lenses converge light rays, hence are called converging lenses.

(*ii*) **Concave or Diverging Lens:** It is thin in the middle and thick at the edges.

All types of concave lenses refract a parallel beam of light outwardly.

As these lenses diverge light rays, hence called *diverging lenses*.

1.7.1 Terms Associated with Spherical Lenses

Some important terms associated with a spherical lens are stated below:

(*i*) **Aperture:** The diameter of the circular edge of the lens, is called the aperture of the lens. In Fig. 1.13, AB is the aperture of the lens.

Brightness of the image is directly proportional to the square of the aperture of the lens.

(*ii*) **Principal Axis:** The straight line passing through the two centres of curvature of the two spherical surfaces of the lens (or though one centre of curvature of one spherical surface and normal to the other plane surface), is called the principal axis of the lens. In Fig. 1.13, XY is the principal axis of the lens.

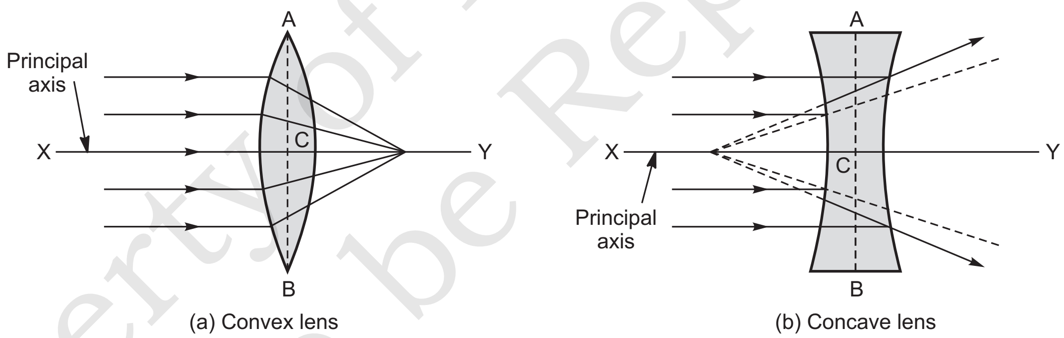


Fig. 1.13. Spherical lenses

(*iii*) **Optical Centre:** It is point on the principal axis of the lens, such that a ray of light passing through it goes undeviated.

In Fig. 1.14, C is the optical centre of the lens.

(*iv*) **First Principal Focus:** It is a point on the principal axis of the lens, such that the rays actually diverging from it (in case of a convex lens) or appear to be going towards it (in case of a concave lens), after refraction from the lens go parallel to the principal axis. Since there are two curved surfaces in a lens, hence there are two principal focus points.

In Fig. 1.14, F_1 is the first principal focus.

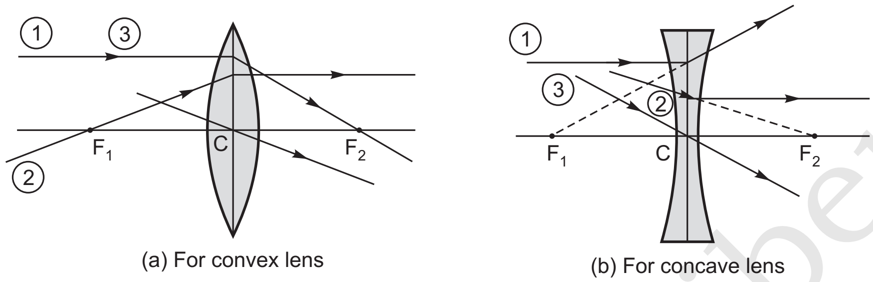


Fig. 1.14. Three special rays

(*v*) **Second Principal Focus:** It is a point on the principal axis of the lens, such that the rays incident on the lens parallel to principal axis after refraction from the lens, actually meet at this point (in case of a convex lens) or appear to come from it (in case of a concave lens).

In Fig. 1.14, F_2 is the second principal focus.

(*vi*) **Focal Length:** The distance between the optical centre of the lens and the principal focus (first or second) of the lens, is called focal length of the lens. It is represented by the symbol *f*.

In Fig. 1.14, $F_1C = CF_2 = f$.

Focal length of a convex lens is positive and focal length of a concave lens is negative.

(*vii*) **Focal Plane:** A plane passing through the principal focus and perpendicular to the principal axis of the lens is called focal plane.

1.8 LOCATION OF IMAGES FORMED BY THIN LENSES USING RAY DIAGRAM METHOD

1.8.1 Rules for Image Formation

When an object is kept in front of a convex lens, an image is formed. This image can be real or virtual depending on the position of the object. However, image is formed at a point where at least two rays after refraction meet or appear to meet. For finding the position and nature of the image, following three rules can be used:

Rule 1. A ray of light incident on the lens parallel to principal axis, (Ray 1), after refraction from the lens, actually passes through its focus (in case of a convex lens) or appears to come from its focus (in case of a concave lens). [Object at infinity, image at focus]

Rule 2. A ray of light incident on the lens through its principal focus (in case of a convex lens) or in direction of principal focus (in case of a concave lens) after refraction from the lens goes parallel to the principal axis. (Ray 2) [Object at focus F_1, image at infinity]

Rule 3. A ray of light incident on the lens and passing through the optical centre, passes undeviated through the lens. (Ray 3)

These special rays are very useful in drawing ray diagrams in different cases.

1.8.2 Ray Diagram

It is a diagram, in which rays are shown coming from a point on the object and falling on a lens and after refraction from the lens are shown either meeting at a point or appearing to diverge from a point, forming its real or virtual image.

If the object is big, it can be divided into many points and point to point images are obtained. Combining the point images, the image of the whole object is obtained.

Real point images produce real image and virtual point image produce virtual image of the complete object.

In Fig. 1.15, *XY* represents principal section of a convex lens. It is taken as a plane sheet due to its small thickness and small aperture.

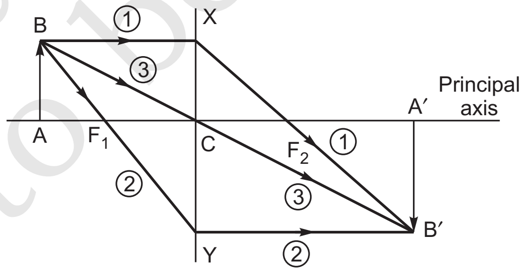


Fig. 1.15. Convex lens—refracting surface towards left

(*a*) Any two of the three special rays are taken to obtain a single point image.

(*b*) Since lenses used are supposed to be thin and have a small aperture, their surfaces can be taken as plane and their principal sections can be represented by a straight line.

AB is a real object having bottom A on the principal axis and top B upwards. Three special rays are shown coming from top B, incident on the lens. After refraction they actually meet at a point B′, which becomes real image of B. A′ lies perpendicularly below B′, on the principal axis. A′ must represent the image of bottom A of the object. A′B′ represents the real image of complete object AB.

(*c*) For small distances and sizes involved, the ray diagram can be drawn on same scale. For bigger distances and sizes, the diagram has to be drawn on a chosen scale.

1.8.3 Location of Image Formed by Convex (Converging) Lenses

Nature of the image formed by a convex lens depends on the position of the object placed in front of the lens. Different positions of the object from the optical centre of the lens produce different types of images. Object can be placed,

(*i*) at infinity (*ii*) beyond 2F
(*iii*) at 2F (*iv*) between F and 2F
(*v*) at F and (*vi*) between F and optical centre.

Different cases are as given ahead with their ray diagrams.

Case 1. *Object at Infinity*

(*i*) *A point object lying on the principal axis:* When a point object is at a large distance from the lens, we usually say that object is at infinity. Since the object is far away from the lens, hence it has not been shown in Fig. 1.16 (*a*). When object is a point object lying on the principal axis, the rays come parallel to the principal axis and after refraction from the lens, actually meet at the second principal focus F_2.

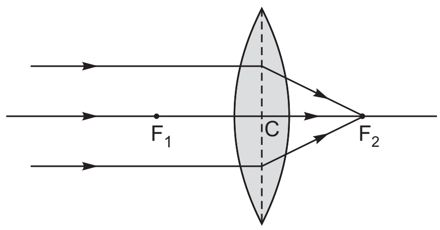


Fig. 1.16 (*a*) Convex lens point object at infinity, image at focus

The image is formed at focus F_2. It is real and point-sized. [Fig. 1.16 (*a*)]

(*ii*) *A big size object with its foot on the principal axis:* When an extended object lies at a large distance on the principal axis, we usually say that object is at infinity. As the object is at infinity, hence it is not shown in Fig. 1.16 (*b*). All the rays coming from a point on the object are parallel and incident on the lens. Ray AD and BC are coming from the same point of the object. Ray AD when passes through the lens, gets refracted along DB′ and ray BC after passing through the optical centre of the lens goes straight along CB′. Thus B′ is the image of the top point of the extended object. If we draw B′A′ perpendicular to the principal axis of the lens, we see that A′B′ is the image of the object placed at infinity.

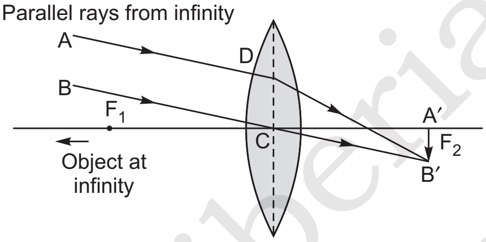


Fig. 1.16 (*b*) Convex lens: big size object at infinity, image at focus

Thus image is formed at the second principal focus F_2. It is real, inverted and diminished (smaller in size than the object).

Case 2. *Object at distance more than twice the focal length.*

The object AB forms its image A′B′ between *f* and 2*f*.

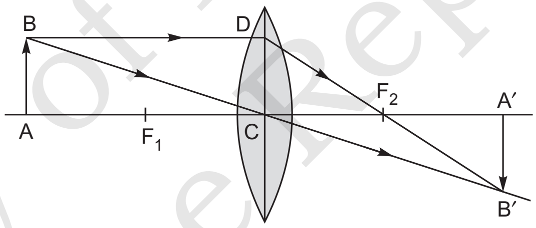


Fig. 1.16 (*c*) Convex lens : object beyond 2*f*, image between *f* and 2*f*

The image is real, inverted and diminished (smaller in size than the object) [Fig. 1.16 (*c*)].

Case 3. *Object at a distance twice the focal length of the lens.*

The real object AB has its real image A′B′ formed at distance 2*f*.

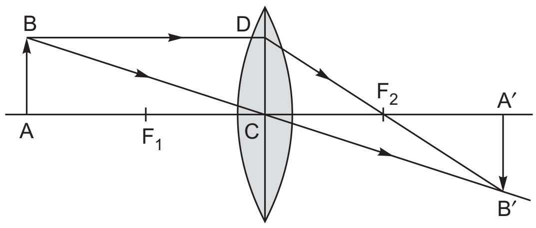


Fig. 1.16 (*d*) Convex lens: object at distance 2*f*, image at distance 2*f*

The image is real, inverted and is of the same size as the object [Fig. 1.16 (*d*)].

Case 4. *Object at a distance more than focal length and less than twice the focal length.*

The real object AB has its image A′B′ formed beyond distance 2*f*.

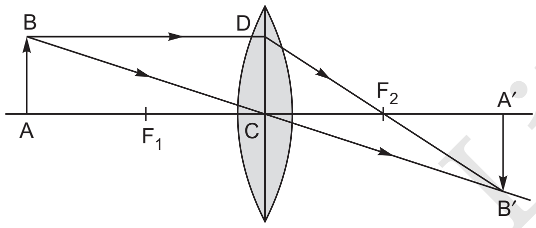


Fig. 1.16 (*e*) Convex lens: object between *f* and 2*f*

The image is real, inverted and enlarged (bigger in size than the object) [Fig.; 1.16 (*e*)]

Case 5. Objective at focus

In such a case, the image is imaginary, inverted (refracted rays go downward), at infinity and must have very large size [Fig. 1.16 (*f*)].

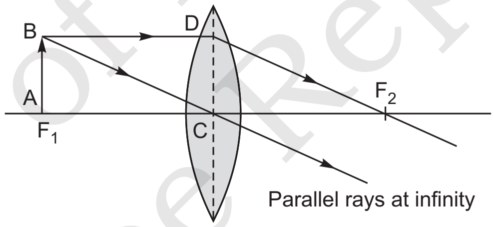


Fig. 1.16 (*f*) Convex lens: object at focus, image at infinity

Case 6. *Object between focus and optical centre*

Real object AB has its image A′B′ formed in front of the lens. The image is virtual, erect and enlarged [Fig. 1.16 (*g*)].

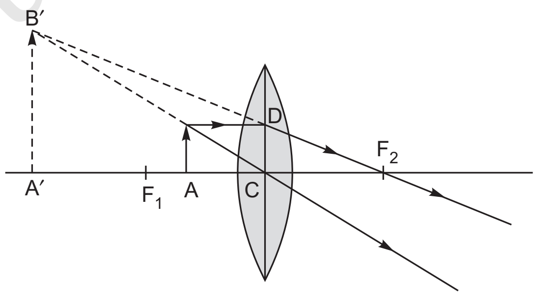


Fig. 1.16 (*g*) Convex lens: object between focus and optical centre, image in front of the lens

The results of all the cases can be given in Table 1.3.

Table 1.3 Image formed by a convex lens for different positions of the object

Case	Object Position		Image	
		Position	**Nature**	**Size**
1.	(*i*) Point object at infinity on the principal axis	at focus	real	Point
	(*ii*) Big object at infinity	at focus	real, inverted	Diminished
2.	At distance more than $2f$	between f and $2f$	real, inverted	diminished
3.	At distance $2f'$	at $2f$	real, inverted	same as object
4.	At distance between f and $2f$	beyond $2f$	real, inverted	enlarged
5.	At focus	at infinity	real, inverted	enlarged
6.	Between focus and optical centre	in front of the lens (on the left side)	virtual, erect	enlarged

1.8.4 Location of Images Formed by Concave (Diverging) Lens

Case 1. *Object at Infinity*

(*i*) A point lying on the principal axis.

Rays come parallel to the principal axis and after refraction from the lens, appear to come from the first principal focus F_1. The image is formed at focus F_1. It is virtual and point-sized [Fig. 1.17 (*a*)].

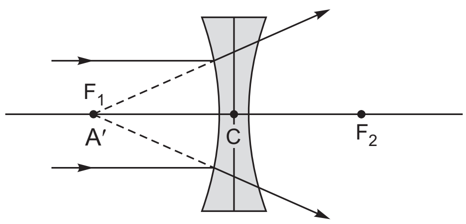


Fig. 1.17 (*a*) Concave lens: point object at infinity, image at focus

Application. Sun light diverging lens.

(*ii*) A big size object with its foot on the principal axis.

Parallel rays come inclined to the principal axis. Image of foot is formed at the focus.

The image is formed at the first principal focus, F_1. It is virtual-erect and diminished [Fig 1.17 (*b*)].

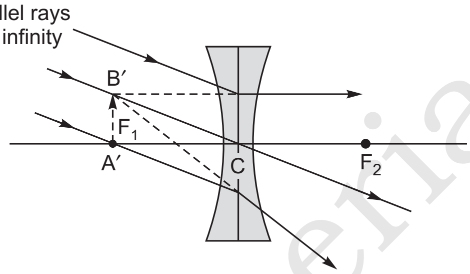


Fig. 1.17 (*b*) Concave lens: point object at infinity, image at focus

Applications.

1. Correcting myopia.
2. Galileo telescope eyepiece (normal adjustment)

Case 2. *Object at a Finite Distance*

Real object AB has its image A′B′ formed between the principal focus and optical centre C.

The image is virtual-erect and diminished [Fig. 1.17 (*c*)].

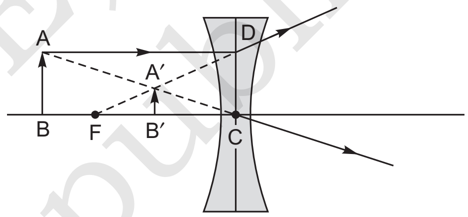


Fig. 1.17 (*c*) Concave lens: Object beyond 2F, image between F and C

The results of all the cases can be given in the Table 1.4.

Application. Galileo telescope eyepiece (near adjustment).

Table 1.4 Image formed by a concave lens for different positions of the object

Case	Object position		Image	
		Position	**Nature**	**Size**
1.	(*i*) Principal axis (*ii*) Big object at infinity	at focus at focus	virtual virtual, erect	point diminished
2.	At finite distance	between F and C	virtual, erect	diminished

1.9 LENS FORMULA

Equation which gives us the relation between distance of the object from the optical centre of the lens (*u*), distance of the image from the optical centre of the lens (*v*) and focal length of the lens (*f*) is,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This equation is called lens formula.

While arriving at this relation, following assumptions are made:

(*i*) The lens is thin.

(*ii*) The lens has a small aperture.

(*iii*) The object lies close to the principal axis.

(*iv*) The incident rays make small angle with the lens surface or the principal axis.

1.9.1 Sign Conventions for Lenses

It is a convention which fixes the signs of different distances to be measured. The sign convention to be followed is the **New Cartesian** sign convention. It gives the following rules:

1. Object is placed on the left hand side of the lens so that incident ray goes from left to right.
2. All distances are measured from the optical centre of the lens.
3. The distances measured in the same direction as the direction of incident light, are taken as positive.
4. The distances measured in the direction opposite to the direction of incident light, are taken as negative.
5. Distance measured upward and perpendicular to the principal axis, are taken as positive.
6. Distance measured downward and perpendicular to the principal axis, are taken as negative.

Object and image lying on the axis in front of the lens, lie on the left of the lens. Their distances are taken negative.

Objects and images, lying on the axis behind the lens, lie on the right of the lens. Their distances are taken positive.

Height of objects and images lying on the axis with their top upward, are taken as positive. Height of objects and images, lying on the axis with their top downward are taken as negative.

In short

Right $\leftrightarrow$ positive,

Left $\leftrightarrow$ negative

Upward $\leftrightarrow$ positive,

Downward $\leftrightarrow$ negative

These facts are shown in Fig. 1.18. Object is on the left of the lens LM.

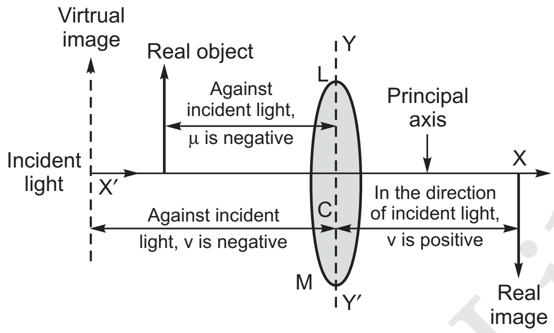


Fig. 1.18. The new cartesian sign convention for refraction by spherical lens

1.9.2 Linear Magnification

Size of the image formed by a lens depends on the position of the object from the lens.

Size of the image can be smaller than or equal to or greater than the size of the object. Size of the image relative to the object is given by linear magnification.

Linear magnification produced by a lens is equal to the ratio of the distance of image from the optical centre (v) to the distance of the object from the optical centre (u)

i.e., Magnification, $m = \frac{v}{u}$

If m is positive, the image is virtual and erect and if m is negative, the image is real and inverted.

If $m > 1$, the image is magnified, if $m < 1$, the image is diminished and if $m = 1$, the size of the image is same as that of the object.

1.10 POWER OF LENS

Power of a lens is the measure of its degree of convergence or divergence of light rays falling on it.

It is the capacity or the ability of a lens to deviate (converge or diverge) the path of rays passing through it. Power of a lens is also defined as the reciprocal of the focal length of the lens when expressed in metres.

It is represented by the symbol P.

A lens having small focal length, focuses a parallel beam of light at near point. Its converging or diverging capability is more.

Hence, it is said to have more power.

Thus, $$\text{Power } (P) \propto \frac{1}{\text{Focal length } (f) \text{ in metres}}$$

i.e., $$\boxed{P = \frac{1}{f \text{ (metre)}}}$$

The *SI* unit for power of a lens is dioptre (*D*): $1D = 1m^{-1}$. The power of a lens of focal length of 1 metre is one dioptre. Power of a lens is positive for a converging lens and negative for a diverging lens.

Power of a converging (convex) lens is positive and that of a diverging (concave) lens is negative.

When two lenses are placed in contact in such a way that their principal axis is same, their combination can be treated as a single lens.

If focal lengths of the individual lenses are f_1, f_2, the focal length (f) of the combination is given by, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

In terms of power of the lens, we can say that the power of the combination of a number of thin lenses placed in contact is equal to the algebraic sum of the power of the individual lenses.

So, $$P = P_1 + P_2$$

1.11 DEFECTS OF VISION AND THEIR CORRECTION

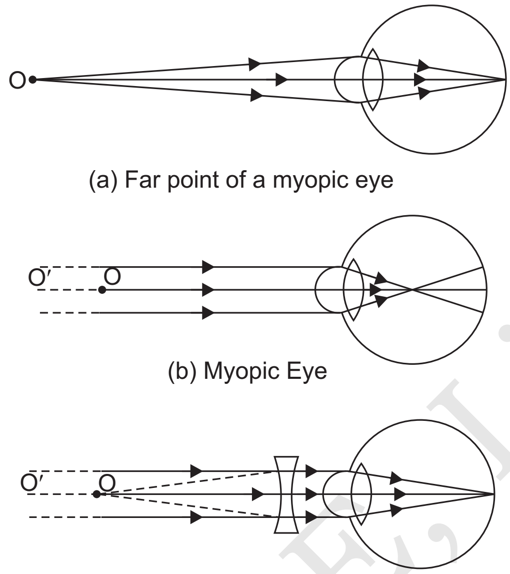
There are mainly three common refractive defects of vision. These are as follows:

Myopia or Near-sightedness: A person with myopia can see nearby objects clearly but cannot see distance objects distinctly. In other words, the far point shifts towards the eye. It is no longer at infinity. In a myopic eye, the image of a distant object is formed in front of the retina and not at the reitna itself.

Cause:

(*i*) Excessive curvature of the eye lens.

(*ii*) Enlongation of the eye-ball.



(a) Far point of a myopic eye

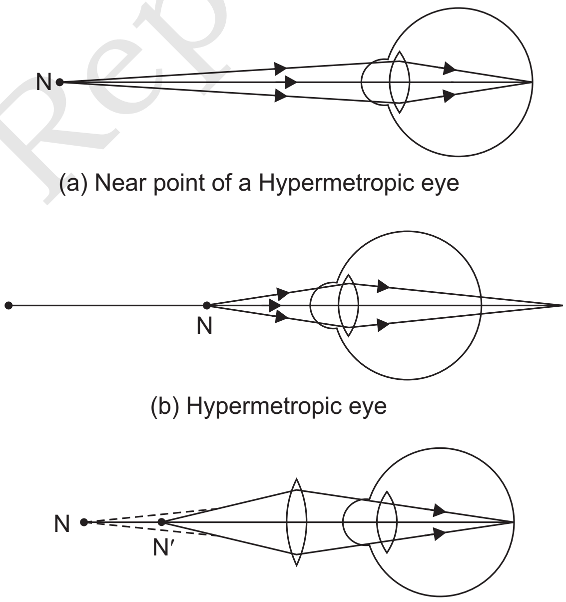
(b) Myopic Eye

(c) Correction of myopia

Fig. 1.19. Chromatic aberration

Correction: It can be corrected by using a concave lens of a suitable power.

Hypermetropia: It is also known as far-sightedness. A person with hypermetropia can see distant objects clearly but cannot see nearby objects distinctly. The nearpoint, for the person is further away from the normal near point (25 cm). The light rays from a closeby object are focused at a point behind the retina.



(a) Near point of a Hypermetropic eye

(b) Hypermetropic eye

(c) Correction for Hypermetropic eye

Fig. 1.20

Cause of Hypermetropoia:

1. The focal length of the eye-lens is too long.
2. The eye ball has become too small.

Correction: This defect can be corrected by using a convex lens of appropriate power.

Presbyopia: The power of accomodation of the eye usually decreases with ageing. For most people, the rear point gradually recedes away. They find it difficult to see nearby objects comfortably and distinctly without correction eye-glasses. This defect is called presbyopia. It arises due to the gradual weakening of the ciliary muscles and diminishing flexibility of the eye lens. Sometimes, a person may suffer from both myopia and hypermetropa. Such people often required bi-focal lenses. It consists of both concave and convex lenses.

1.12 LENS DEFECTS AND THEIR CORRECTIONS

There are two main types of defects in a lens—chromatic aberration and spherical aberration.

1.12.1 Chromatic Aberration

Chromatic aberration, also known as "colour fringing" or "purple fringing", is a common optical problem that occurs when a lens is either unable to bring all wavelengths of colour to the same focal plane, and/ or when wave-lengths of colour are focused at different positions in the focal plane. Chromatic aberration is caused by lens dispersion, with different colours of light travelling at different speeds while passing through a lens. As a result, chromatic aberration occurs when different wavelengths of colour do not converge at the same point after passing through a lens, as illustrated below.

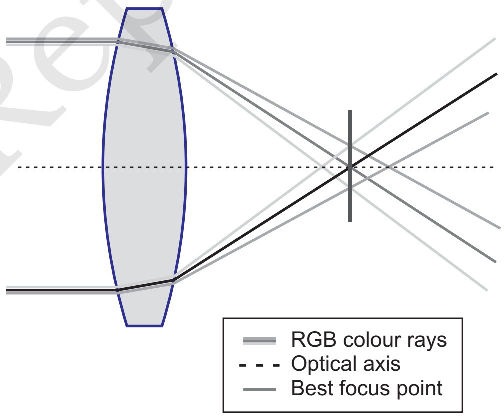


Fig. 1.21. Chromatic aberration

Correction of Chromatic Aberration

Chromatic aberration can be minimized by combining a convex lens of crown glass and a concave lens of flint glass in such a way that dispersion of light produced by the convex lens is neutralized by the concave lens. Such a combination of lenses is called an " Achromatic Lens". In high class cameras and optical instruments, a complicated combination of lenses is used.

1.12.2 Spherical Aberration

In spherical aberration, parallel light rays that pass through the central region of the lens focus farther away than light rays that pass through the edges of the lens. The result is many focal points, which produce a blurry image. To get a clear image, all rays need to focus at the same point.

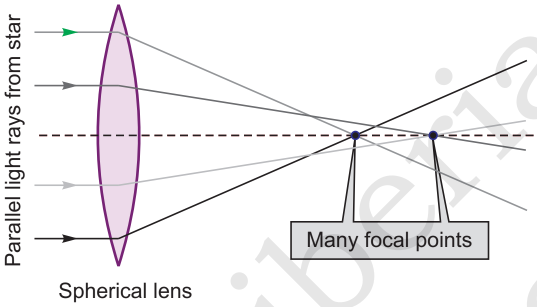


Fig. 1.22. Spherical Aberration

Correction of Spherical Aberration

This defect is removed by using a complicated lens made by combining lenses of different shapes.

1.13 DISPERSION OF LIGHT THROUGH A GLASS PRISM

Materials required:

Prism, white light and a dark room where the refraction can be clearly seen.

Procedure:

- Take the prism and keep it on the table.
- Focus a beam of light from a light source to the prism.

What do you observe? You will observe that the white light splits into 7 colours. This shows dispersion of light.

Dispersion is the splitting up of white light into seven colours on passing through a transparent medium like a glass prism.

It has been known for a long time that when a narrow beam of sunlight usually called white light, is incident on a glass prism, the emergent light is seen to be

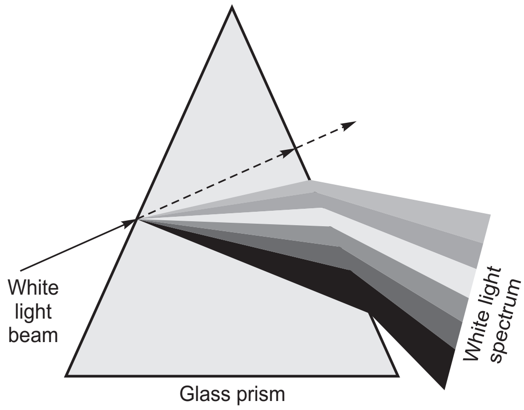


Fig. 1.23. Dispersion of sunlight or white light on passing through a glass prism. The relative deviation of different colours shown is highly exaggerated.

consisting of several colours. There is actually a continuous variation of colour, but broadly, the different component colours that appear in sequence are: violet, indigo, blue, green, yellow, orange and red (given by the acronym VIBGYOR). The red light bends the least, while the violet light bends the most (Fig. 1.23).

The phenomenon of splitting of light into its component colours is known as *dispersion*. The pattern of colour components of light is called the spectrum of light.

Example. A concave lens has focal length of 15 cm. At what distance should the object from the lens be placed so that it forms an image at 10 cm from the lens? Also, find the magnification produced by the lens.

Solution. A concave lens always forms image on the same side of the object.

Image-distance $v = -10$ cm; Focal length $f = -15$ cm;

Object–distance $u = ?$

Since $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$\frac{1}{u} = \frac{1}{-10} - \frac{1}{(-15)} = -\frac{1}{10} + \frac{1}{15}$$

$$\frac{1}{u} = \frac{-3+2}{30} = \frac{1}{-30} \quad \text{or} \quad u = -30 \text{ cm}$$

Thus, the object-distance is 30 cm.

Magnification, $m = v/u$

$$m = \frac{-10 \text{ cm}}{-30 \text{ cm}} = \frac{1}{3} = +0.33$$

Example. An object is placed perpendicular to the principal axis of a convex lens of focal length 10 cm. The distance of the object from the lens is 15 cm. Find the location of the image.

Solution. Focal length, $f = +10$ cm

Object-distance, $u = -15$ cm

Image-distance, $v = ?$

Since $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{(-15)} + \frac{1}{10} = -\frac{1}{15} + \frac{1}{10}$$

$$\frac{1}{v} = \frac{-2+3}{30} = \frac{1}{30} \quad \text{or} \quad v = 30 \text{ cm}$$

The positive sign of v shows that the image is formed at a distance of 30 cm on the other side of the optical centre.

Example. A magician during a show makes a glass lens with $n = 1.47$ disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

Solution. The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This gives $1/f = 0$ or $f \to \infty$. The lens in the liquid will act like a plane sheet of glass. No, the liquid is not water. It could be glycerine.

Example. If $f = 0.5$ m for a glass lens, what is the power of the lens?

Solution. $P = 1/f = 1/0.5 = 2$ dioptre

REVIEW EXERCISE

A. MULTIPLE CHOICE QUESTIONS (MCQs)

1. Bending of a ray of light, when it enters obliquely from one medium to other is called

(*a*) reflection (*b*) refraction

(*c*) dispersion (*d*) interference.

2. The relation, $\frac{\sin i}{\sin r} = n$, is called

(*a*) Snell's law (*b*) Newton' law

(*c*) Joule's law (*d*) Boyle's law.

3. When light enters from first rarer medium to a second denser medium, then $^1\mu_2$ has value

(*a*) < 1 (*b*) 1

(*c*) > 1 (*d*) no definite relation

4. For light going from air to water, ${}^{a}n_{w} = 4/3$. Then ${}^{w}n_{a}$ has value

(*a*) $\frac{16}{9}$ (*b*) $\frac{3}{2}$

(*c*) 1 (*d*) $\frac{3}{4}$.

5. In case of refraction of light from a rectangular glass slab, if i = angle of incidence and e = angle of emergence, then

(*a*) $e < i$ (*b*) $e = i$

(*c*) $e > i$ (*d*) no definite relation.

6. A lens is thick in the middle and thin at the edges. The lens is

(*a*) concave (*b*) convex

(*c*) plane (*d*) prism.

7. A lens is thin in the middle and thick at the edges. The lens is

(*a*) concave (*b*) convex

(*c*) plane (*d*) prism.

8. A lens converges rays of light. The lens is

(*a*) plane (*b*) prism

(*c*) concave (*d*) convex.

9. A lens diverges rays of light. The lens is

(*a*) plane (*b*) prism

(*c*) concave (*d*) convex.

10. A ray of light passes undeviated through a point on the principal axis. The point is

(*a*) focus (*b*) centre of curvature

(*c*) optical centre (*d*) no where.

11. For drawing a ray diagram, we take light in form of

(*a*) rays (*b*) pencil

(*c*) beam (*d*) bunch.

12. For studying refraction through a lens, we keep the lens with its refracting surface towards

(*a*) right (*b*) left

(*c*) up (*d*) down.

13. The relation between object distance u, image distance v and focal length f is called

(*a*) lens formula (*b*) mirror formula

(*c*) object formula (*d*) image formula.

14. In case of erect object having inverted image, linear magnification is
    (*a*) positive (*b*) negative
    (*c*) zero (*d*) no definite sign.
15. In case of erect object, having erect image, linear magnification is
    (*a*) positive (*b*) negative
    (*c*) zero (*d*) no definite sign.

B. FILL IN THE BLANKS

1. In refraction, a ray of light .... when it enters obliquely in some other medium.
2. The quantity, $\frac{\sin i}{\sin r} = n$ is called the .... of the medium.
3. If $^{a}n_{g} = 3/2$, then $^{g}n_{a}$ = ...
4. In case of refraction from a rectangular glass slab, angle of emergence *e* is .... angle of incidence *i*.
5. A convex lens .... rays.
6. Rays are diverged by a .... lens.
7. A ray of light passing through the optical centre of a lens goes ....
8. Object distance for an object on the left side of a lens is ....
9. Image distance for the image on the right of the lens is ....
10. Magnification is negative for an erect object and .... image.

C. VERY SHORT ANSWER TYPE QUESTIONS

1. What is meant by refraction of light?
2. Where does refraction of light occur, in the other medium or at the boundary separating the media?
3. Is speed of light in vacuum a fundamental constant?
4. What do you mean by an optical medium?
5. Where does light travel faster, in optically denser or in optically rarer medium?
6. Out of air, water and glass which is optically densest medium?
7. What does optical density of a medium signify? Is optical density of a medium same as mass density of the medium?
8. What is the minimum value of refractive index?

9. Is refractive index of a medium always constant?

10. Refractive index of medium 2 w.r.t. medium 1 is reciprocal of refractive index of medium 1 w.r.t. medium 2. Prove it.

D. SHORT ANSWER TYPE QUESTIONS

1. Identify the device used as a spherical mirror or lens in following cases, when the image formed is virtual and erect in each case.

(*a*) Object is placed between device and its focus, image formed is enlarged and behind it.

(*b*) Object is placed between the focus and device, image formed is enlarged and on the same side as that of the object.

2. Why does a light ray incident on a rectangular glass slab immersed in any medium emerges parallel to itself? Explain using a diagram.

3. A pencil when dipped in water in a glass tumbler appears to be bent at the interface of air and water. Will the pencil appear to be bent to the same extent, if instead of water we use liquids like, kerosene or turpentine. Support your answer with reason.

4. How is the refractive index of a medium related to the speed of light? Obtain an expression for refractive index of a medium with respect to another in terms of speed of light in these two media?

5. Refractive index of diamond with respect to glass is 1.6 and absolute refractive index of glass is 1.5. Find out the absolute refractive index of diamond.

E. LONG ANSWER TYPE QUESTIONS

1. How much time will light take to cross 2 mm thick glass pane if refractive index of glass is 3/2?

2. A concave lens of focal length 15 cm forms an image 10 cm from the lens. How far is the object placed from the lens? Draw the ray diagram.

3. A pond of depth 20 cm is filled with water of refractive index 4/3. Calculate apparent depth of the tank when viewed normally.

4. An object of size 7.0 cm is placed at 27 cm in front of a concave mirror of focal length 18 cm. At what distance from the mirror, should a screen be placed, so that a sharp focussed image can be obtained? Find the size and nature of the image?

5. A 2.0 cm tall object is placed perpendicular to the principal axis of a convex lens of focal length 10 cm. The distance of the object from the lens is 15 cm. Find the nature, position and size of the image. Also find the magnification.

PERIOD-2

TOPIC

2

Direct Current (DC) Electricity

2.1 PRIMARY AND SECONDARY CELLS

A Cell or Battery is an electrical component that converts Chemical Energy into Electrical Energy. Both Cell and Battery are the same combination of electrochemical Cells. The Cell is a simple and small unit. Many Cells make up a Battery and therefore, the Battery is called a cluster of Cells. The Cell is relatively small in size than the Battery.

There are mainly two types of Cells. This is briefly discussed below.

Primary Cell

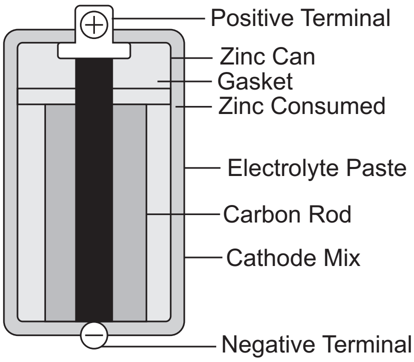
In there Cells, electrical energy is generated using an irreversible Chemical reaction. Rechargeable Cells come out slowly. They are very intertwined. Because of the lack of fluid in the Cell, it is also called a dry Cell. Although easy to use, they cannot be reused. In general, they have very high internal opposition.

Secondary Cell

Secondary regenerative Cells convert Chemical Energy into electrical Energy and vice versa. They are charged again by the power supply. There is low internal resistance in this Cell. They are relatively inexpensive to use than the main Cell. They cost more than a basic Cell.

The Difference Between a Basic and a Secondary Cell

Primary Cells are the only ones that can be charged and need to be discarded after the end of the life span, while Secondary Cells need to be recharged once the charging is complete. Both types of batteries are widely used in a variety of applications and these Cells vary in size and material.



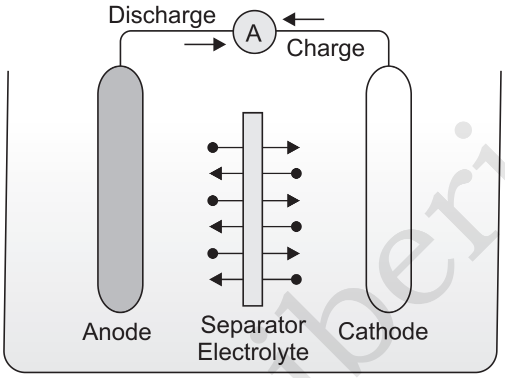


Fig. 2.1

Difference between Primary and Secondary Cell

S.No.	Primary Cell	Secondary Cell
1.	These cannot be recharged again after getting discharged once.	These can be recharged easily.
2.	These are cheap or low cost.	These are expensive compared to Primary Cell.
3.	These are easy to use.	These are difficult to use in comparison to Primary Cell.
4.	These can be used only once.	These can be used more than once.
5.	In these Cells irreversible reactions occur.	In these Cells reversible reaction occurs.
6.	These have a lower self-discharge rate.	These have a higher self-discharge rate.
7.	These are used in torch and other portable devices as they produce electric current immediately.	These are used in inverters and automobiles.
8.	These Cells don't require regular maintenance and can be disposed of easily after use.	These Cells require regular maintenance.
9.	They have a low or small lifetime.	They have a high lifetime.
10.	Examples of these Cells are dry Cells, Daniel Cells etc.	Examples of these Cells are lead-acid Cell, nickel – iron Cell etc.

2.2 FUEL AND SOLAR CELLS

Plants split water into H^+ and O_2 using natural *catalysts* that are renewed every 30 minutes. We can already do the same thing very easily using electricity and metal electrodes in a process called *electrolysis* (water splitting) but this is very expensive in both energy and materials cost. To store solar energy in a H_2 fuel on a large scale to meet the Terawatt challenge, we need to be able to split water with renewable energy or a solar driven process using inexpensive materials. This is indeed what a number of researchers startup and established companies around the world are currently working hard to achieve.

Yet, if H_2 is a fuel that's storing energy, how do we get the energy back out?

That's easy using fuel cells that produce electricity by doing exactly the opposite of splitting water-reacting H_2 and O_2 to form water again as in the animation below that shows the operation of one cell in a fuel cell stack. So fuel cells run by consuming hydrogen, a 'zero-emission' fuel and produce only water and electricity.

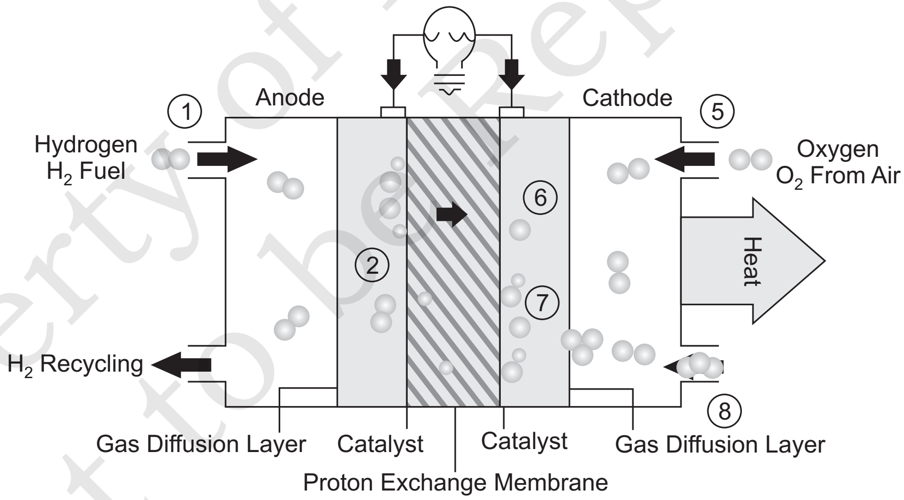


Fig. 2.2

2.3 ELECTRIC CURRENT

(i) *The branch of physics which deals with the motion of charges is called* ***current electricity.***

In metallic conductors such as silver, copper, aluminium etc., the electrons in the outermost orbits of the atoms are loosely bound to their respective atoms. These electrons can be made easily free. These are known as *free electrons.* Since the electronic current in a conductor is due to the motion of free electrons, therefore, these electrons are also known as *conduction electrons.* In other words, the free electrons act as charge carriers in metallic conductors.

The flow of free charges in a conductor constitutes electric current.

(ii) Electric current

The ***electric current*** *is defined as the charge flowing through any section of the conductor in one second.* In other words, it is the rate of flow of electric charge through any section of the conductor.

If the rate of flow of charge is independent of time, then the electric current is said to be steady. In a steady flow of charge, if a charge q crosses any section of the conductor in time t, then the current flowing through the conductor is given by,

$$\boxed{I = \frac{q}{t}}$$

If the rate of flow of charge varies with time, then the current at any time *i.e.*, instantaneous current is given by

$$\boxed{I = \frac{dq}{dt}}$$

where dq is the extremely small amount of charge crossing any section of the conductor in small time dt.

If n be the number of conduction electrons crossing a certain cross-section of the conductor in time t,

then $$I = \frac{ne}{t} \quad \text{or} \quad \boxed{n = \frac{It}{e}}$$

(iii) Current-time Graphs

Graph of Fig. 2.3 represents a steady current.

Steady current is that current which does not vary with time.

The graphs (*a*), (*b*) and (*c*) of Fig. 2.4 represent varying currents.

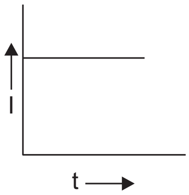


Fig. 2.3

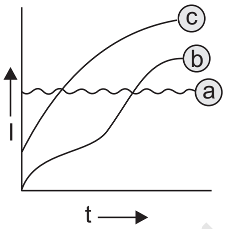


Fig. 2.4

Varying current is a current whose magnitude varies with time.

Graph of Fig. 2.5 shows alternating current. *An alternating current is a current whose magnitude changes continuously with time and direction changes periodically.*

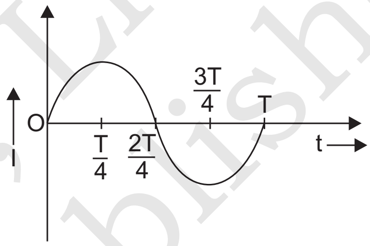


Fig. 2.5

(iv) Measurement of Current

In actual practice, the measurement of electric current does not include individual measurements of charge and time. The electric current is generally measured by its magnetic effect, chemical effect, heating effect etc.

(v) Convention for the Direction of Current

In the year 1820, Ampere put forward a **convention for the direction of electric current.** According to this convention, *the direction of electric current is the direction in which a positive charge would move under the action of an electric field.* In a metallic conductor, the positive charges cannot move. The electric current is wholly due to the movement of the negative charges *i.e.*, free electrons. *But a negative charge moving in one direction is equivalent to an equal amount of positive charge moving in the opposite direction.* So, the direction of conventional current in a metallic conductor is opposite to that in which the free electrons move (Fig. 2.6).

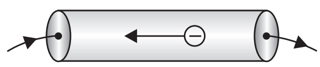


Fig. 2.6. Direction of current

2.4 SI UNIT OF ELECTRIC CURRENT

The **SI unit** of electric current is **ampere** (A).

The ***current*** *flowing in a conductor is said to be one ampere if one coulomb of charge flows across any of its cross-section in one second.*

$$\therefore \qquad 1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

One ampere may also be defined as that constant current which when flowing in two straight parallel conductors of infinite length and of negligible cross-sectional area and placed one metre apart in vacuum would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

Example 1: *Calculate the number of electrons crossing a given cross-section in 1 second to constitute a current of 1 A.*

Solution: $I = \frac{q}{t} = \frac{ne}{t}$

or $I = \frac{n}{t} \times 1.6 \times 10^{-19}$ or $\frac{n}{t} = \mathbf{6.25 \times 10^{18}\ s^{-1}}$

Example 2: *A current of 0.50 ampere is passing through a $CuSO_4$ solution. How many Cu^{++} ions will be deposited on cathode in 10 second?*

Solution: Current, I = 0.50 A ; Time, t = 10 s

Each Cu^{++} ion carries charge of two protons. If n be the number of copper ions, then charge,

$$q = n \times 2e$$

Now, $I = \frac{q}{t}$ or $q = It$ or $n \times 2e = It$

or $$n = \frac{It}{2e} = \frac{0.50\ A \times 10\ s}{2 \times 1.6 \times 10^{-19}\ C} = \mathbf{1.5625 \times 10^{-19}}$$

2.5 OHM'S LAW

Statement. ***Physical conditions such as temperature, mechanical strain etc., remaining the same, the electric current flowing through a conductor is directly proportional to the potential difference across the two ends of the conductor.***

Imagine a conductor through which a current I is flowing and let V be the potential difference between the ends of the conductor. Then, according to Ohm's law,

$$V \propto I \quad \text{or} \quad V = RI$$

where the constant of proportionality R is called the resistance of the conductor.

Since the current I is proportional to the potential difference V therefore the graph between V and I is a straight line (Fig. 2.7).

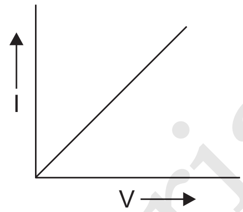


Fig. 2.7. V-I graph for an Ohmic conductor

Example 3: *In a discharge tube, the number of hydrogen ions (i.e., protons) drifting across a cross-section per second is 1.0 × 10^18, while the number of electrons drifting in the opposite direction across another cross-section is 2.7 × 10^18 per second. If the supply voltage is 230 V, what is the effective resistance of the tube ?*

Solution:

$$I = \frac{(1.0 \times 10^{18} + 2.7 \times 10^{18}) \times 1.6 \times 10^{-19}}{1} \text{A} = 0.592 \text{ A}$$

$$\text{Resistance, } R = \frac{V}{I} = \frac{230}{0.592} \Omega = \mathbf{388.5}\ \Omega$$

Example 4: *A copper bar carrying 1000 A has a potential drop of 1 mV along 10 cm of its length. What is the resistance per metre of the bar ?*

Solution:

$$R = \frac{V}{I} = \frac{10^{-3}}{1000} \Omega = 10^{-6}\ \Omega$$

$$\text{Resistance/metre} = \frac{10^{-6} \times 100}{10} \Omega\ m^{-1} = \mathbf{10\ \mu\Omega\ m^{-1}}$$

2.6 ELECTRICAL RESISTANCE

(i) *The* ***resistance*** *of conductor is the opposition offered by the conductor to the flow of electric current through it.*

We know that, $R = \dfrac{ml}{ne^2 A\tau}$ or $R = \dfrac{m}{ne^2\tau} \times \dfrac{l}{A}$ or $\boxed{R = \rho \dfrac{l}{A}}$

where $\rho \left(= \dfrac{m}{ne^2\tau} \right)$ is called the **specific resistance** or **electrical**

resistivity of the material of the conductor. Both n and τ depend on the nature of material of the conductor. So, ρ depends on the nature of material of the conductor and not on the dimensions of the conductor.

(*ii*) Factors affecting resistance. The resistance of a conductor is *directly proportional to the length* of the conductor, provided other factors remain unchanged.

The resistance of a conductor is *inversely proportional to the cross-sectional area* of the conductor, provided other factors remain unchanged.

The resistance of a conductor also depends upon the *nature of material* and *temperature* of the conductor.

(*iii*) Units. We know that, $R = \frac{V}{I}$

So, the resistance of a conductor is the ratio between the potential difference V across the ends of the conductor and the electric current I flowing through the conductor.

The **SI unit** of resistance is **ohm.** It is denoted by the symbol Ω.

The resistance of a conductor is said to be one ohm if a current of one ampere flows through the conductor when a potential difference of one volt is applied across its ends.

$$\textbf{1 ohm} = \frac{\textbf{1 volt}}{\textbf{1 ampere}} \quad \text{or} \quad \mathbf{1\ \Omega = 1\ V\ A^{-1}}$$

One **international ohm** is the resistance of a column of mercury of length 106.3 cm and having an area of cross-section 1 sq. mm at 0°C.

1 international ohm = 1.0006495 ohm.

Larger units of resistance are called kilo ohm and mega ohm.

1 kilo ohm = 1000 ohm and 1 mega ohm = 10^6 ohm

The smaller unit of resistance is micro ohm. 1 micro ohm = 10^{-6} ohm

(*iv*) Dimensional formula.

$$R = \frac{V}{I} = \frac{\text{Work done}}{\text{Charge}} \times \frac{1}{\text{Current}}$$

$$= \frac{\text{Work done}}{\text{Current} \times \text{Time} \times \text{Current}}$$

$$[R] = \frac{[ML^2T^{-2}]}{[A^2T]} = [ML^2T^{-3}A^{-2}]$$

2.7 ELECTRICAL RESISTIVITY (SPECIFIC RESISTANCE)

(*i*) **Definition and explanation.** We know that

$$R = \rho \frac{l}{A} \text{. If } l = 1 \text{ and } A = 1 \text{, then}$$

$$\rho = R.$$

This leads us to the following definition of resistivity.

The ***resistivity*** *of a material is the resistance of a conductor of this material of unit length and unit cross-sectional area. The shape of the cross-section is immaterial.*

The ***resistivity*** *of a material may also be defined as the resistance of a unit cube of that material.*

Those substances which have ***low resistivity*** are conductors of electricity. Metals belong to this category. Conductors are used for transporting electric current without appreciable loss of energy.

Those substances which have ***high resistivity*** are non-conductors of electricity *i.e.,* insulators. When we do not want electric current to flow between two points, then an insulating material such as glass, mica or bakelite is put between those two points. These materials are used to avoid electric shocks.

(*ii*) **Factors affecting resistivity.** We know that, $R = \left[\frac{m}{ne^2\tau}\right]\frac{l}{A}$

Also, $R = \rho\frac{l}{A}$. Comparing, $\boxed{\rho = \frac{m}{ne^2\tau}}$

It follows from here that the resistivity of the material of a conductor depends upon the following factors :

(*a*) Resistivity is inversely proportional to n *i.e.,* the number of free electrons per unit volume. But the value of n depends upon the nature of the material. So, the resistivity of a conductor depends upon the nature of material of the conductor.

(*b*) Resistivity is inversely proportional to τ *i.e.,* the average relaxation time of the free electrons in the conductor. τ decreases with rise of temperature. So, resistivity increases with the increase in temperature of the conductor.

The free electron density and the relaxation time are different for different materials. So, the resistivity for different materials is different.

(*iii*) **Unit.** $R = \rho \frac{l}{A}$ or $\rho = \frac{RA}{l}$

The SI unit of ρ is $\frac{\text{ohm metre}^2}{\text{metre}}$ or ohm metre *i.e.,* Ω m.

(*iv*) **Dimensional formula**

$$\rho = \frac{RA}{l} \;; \quad [\rho] = \frac{[ML^2T^{-3}A^{-2}][L^2]}{[L]} = [ML^3T^{-3}A^{-2}]$$

2.8 ELECTRICAL CONDUCTANCE

The reciprocal of resistance is called ***conductance.*** It is denoted by G.

$$G = \frac{1}{R} = \frac{A}{\rho l}$$

The SI unit of conductance is **ohm^{-1} (Ω^{-1})** or **mho** or **A V^{-1}** or **siemen (S).**

2.9 ELECTRICAL CONDUCTIVITY

As the name suggests, the conductivity of a material is its ability to conduct electric current. It is a measure of the ease with which the current flows through a conductor.

Electrical conductivity is defined as the reciprocal of resistivity. It is denoted by σ.

$$\boxed{\sigma = \frac{1}{\rho}} \quad \text{or} \quad \boxed{\sigma = \frac{ne^2\tau}{m}}$$

The SI unit of conductivity is ohm^{-1} metre^{-1} **(Ω^{-1} m^{-1})** or mho metre^{-1} or siemen metre^{-1} **(S m^{-1}).**

$$[\sigma] = \frac{[L^{-3}][AT]^2[T]}{[M]}; \quad [\sigma] = [M^{-1}L^{-3}T^3A^2]$$

Example 5: *A nichrome wire of resistivity 'ρ' is stretched to make it 10% longer. What is the percentage change in its resistance?*

Solution: We have, $l' = l + \frac{10}{100} l = l + 0.1\, l$

$$= 1.1\, l \quad \text{or} \quad \frac{l'}{l} = 1.1.$$

Since volume is to remain unchanged,

$$\therefore \qquad A'l' = Al \quad \text{or} \quad \frac{A'}{A} = \frac{l}{l'} = \frac{1}{1.1}$$

$$R' = \rho\frac{l'}{A'} = \frac{\rho(1.1\,l)(1.1)}{A} = (1.1)^2 R = 1.21\,R$$

$$\text{Now,} \qquad \frac{R' - R}{R} \times 100 = \frac{1.21\,R - R}{R} \times 100 = \mathbf{21\%}$$

2.10 SERIES COMBINATION OF RESISTORS

Resistors are said to be connected in series if the same current flows through each resistor when some potential difference is applied across the combination.

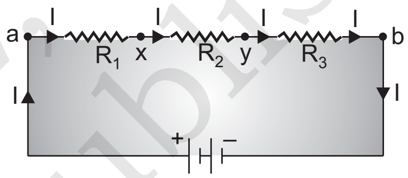


Fig. 2.8. Series combination of resistors

When resistors are connected in series between two points, they provide a single path between the two points.

It is possible to find a single resistor which could replace series combination of resistors in any given circuit and leave unaltered the potential difference between the terminals of the combination and the current in the rest of the circuit. The resistance of this single resistor is called the **equivalent resistance** of the combination.

If R_s be the equivalent resistance and V_{ab} the potential difference between the terminals *a* and *b* of the network, then

$$V_{ab} = IR_s \qquad \ldots(1)$$

where I is the current flowing in the network.

When the resistors are connected in series, the current in each must be the same and equal to the line current I.

$$\therefore \qquad V_{ax} = IR_1,\ V_{xy} = IR_2 \text{ and } V_{yb} = IR_3$$

$$\text{Now,} \qquad V_{ax} + V_{xy} + V_{yb} = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

$$\text{But} \qquad V_{ax} = V_a - V_x,\ V_{xy} = V_x - V_y \text{ and } V_{yb} = V_y - V_b$$

$$\therefore \qquad V_{ax} + V_{xy} + V_{yb} = V_a - V_x + V_x - V_y + V_y - V_b$$

$$= V_a - V_b = V_{ab}$$

$$\therefore \qquad V_{ab} = I(R_1 + R_2 + R_3)$$

$$\text{or} \qquad IR_s = I(R_1 + R_2 + R_3) \qquad \text{[From Eqn. (1)]}$$

$$\text{or} \qquad \boxed{R_s = R_1 + R_2 + R_3}$$

This relation can be generalised for any number of resistances connected in series. So, if n resistances are connected in series, then their equivalent resistance is given by

$$R_s = R_1 + R_2 + R_3 + ... + R_n \qquad \text{or} \qquad \boxed{R_s = \sum_{i=1}^{i=n} R_i}$$

Conclusion. *The equivalent resistance of any number of resistors in series is equal to the sum of their individual resistances.*

2.11 PARALLEL COMBINATION OF RESISTORS

Resistors are said to be connected in parallel if the potential difference across each of them is the same and is equal to the applied potential difference.

Figure 2.9 shows three resistors of resistances R_1, R_2 and R_3 connected in parallel. Each resistor provides an alternative path between the two terminals a and b of the network. Let R_p be the equivalent resistance of the network.

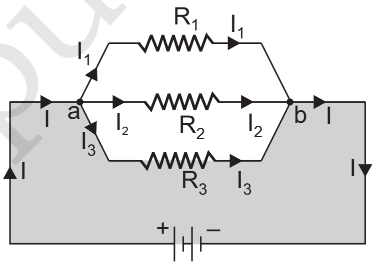


Fig. 2.9. Parallel combination of resistors

$$V_{ab} = IR_p \qquad ...(1)$$

Since the resistors are connected in parallel therefore the potential difference between the terminals of each must be the same and equal to V_{ab}. If I_1, I_2 and I_3 are the currents in resistances R_1, R_2 and R_3 respectively, then

$$I_1 = \frac{V_{ab}}{R_1}, I_2 = \frac{V_{ab}}{R_2} \quad \text{and} \quad I_3 = \frac{V_{ab}}{R_3}.$$

Charge is delivered to the terminal a by the line current I and removed from a by the currents I_1, I_2 and I_3. So, no charge accumulates at a.

$\therefore \quad I = I_1 + I_2 + I_3 \quad \text{or} \quad I = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$

or
$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But
$$\frac{I}{V_{ab}} = \frac{1}{R_p} \qquad \text{[From Eqn. (1)]}$$

$\therefore$
$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

This relation can be generalised for any number of resistances connected in parallel. So, if n resistances are connected in parallel, then

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots.. + \frac{1}{R_n}$$

or
$$\boxed{\frac{1}{R_p} = \sum_{i=1}^{n} \frac{1}{R_i}}$$

Conclusion. *For any number of resistors connected in parallel, the reciprocal of the equivalent resistance is equal to the sum of the reciprocals of their individual resistances.*

2.12 DISTRIBUTION OF ELECTRIC CURRENT IN TWO PARALLEL RESISTANCES

Consider two resistances R_1 and R_2 connected in parallel. The current I will be divided into two parts at the terminal x. Suppose current I_1 flows through R_1 and current I_2 flows through R_2. The currents I_1 and I_2 combine at y to give the original current I. If R be the total resistance between x and y, then

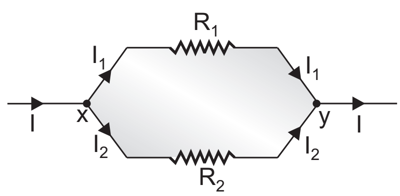


Fig. 2.10. Distribution of current

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

If V be the potential difference between x and y, then

$$V = IR = \frac{IR_1R_2}{R_1 + R_2}. \quad \text{Now,} \quad I_1 = \frac{V}{R_1} = \frac{IR_2}{R_1 + R_2} \qquad ...(1)$$

$$\text{Similarly,} \quad I_2 = \frac{IR_1}{R_1 + R_2} \qquad ...(2)$$

Dividing (1) by (2), we get,

$$\frac{I_1}{I_2} = \frac{IR_2}{R_1 + R_2} \times \frac{R_1 + R_2}{IR_1} \quad \text{or} \quad \boxed{\frac{I_1}{I_2} = \frac{R_2}{R_1}}$$

Conclusion. *The currents divide themselves in the inverse ratio of resistances.*

Example 8: *Find the strength of current flowing through the circuit shown in Fig. 2.11.*

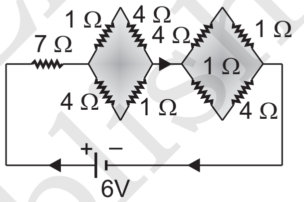


Fig. 2.11

Solution: In the given network, 4 Ω and 1 Ω are in series. This gives 5 Ω. So, the given circuit may be redrawn as shown in Fig. 2.12.

Now, $R = (7 + 2.5 + 2.5)\ \Omega$

$= (7 + 5)\ \Omega = 12\ \Omega$

Current, $I = \frac{V}{R} = \frac{6}{12}$ A

$= \frac{1}{2}$ A = **0.5 A**

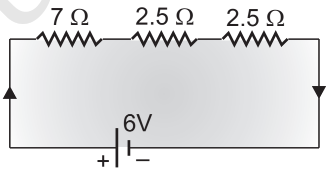


Fig. 2.12

2.13 TEMPERATURE DEPENDENCE OF RESISTANCE

We know that $R = \frac{m}{ne^2\tau} \times \frac{l}{A}$

For a given conductor, $\boxed{R \propto \frac{1}{\tau}}$

When a metallic conductor is heated, the atoms in the metal vibrate with greater amplitude and frequency about their mean positions. Due to increase in thermal energy, the thermal velocities of the free electrons also increase. Consequently, the number of collisions between free electrons and atoms increases. This reduces the relaxation time τ and increases the value of resistance R.

The resistance R_t of a metallic conductor at temperature t°C is given by

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

where α and β are constants whose values vary from metal to metal. If t is not sufficiently large, then

$$R_t = R_0(1 + \alpha t)$$

where α is called the temperature coefficient of resistance of the material of the conductor.

The temperature coefficient of resistance is defined as the change in resistance per unit resistance at 0ºC per degree rise of temperature.

Strictly speaking, α is defined with respect to 0°C. The value of α is different at different temperatures. Temperature coefficient of resistance **averaged over the temperature range** t_1°C to t_2°C is given by,

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$

For **metals,** (silver, copper), α is positive *i.e.,* the resistance increases with increase in temperature. For most of the pure metals, the value of α is of the order of $\frac{1}{273}$ °C^{-1}.

2.14 TEMPERATURE DEPENDENCE OF RESISTIVITY

(i) Metallic Conductors. We know that

$$\rho = \frac{m}{ne^2\tau} \quad \ldots(1)$$

Resistivity is inversely proportional to relaxation time. When the temperature of a metallic conductor is increased, the relaxation time is decreased on account of increased atomic vibrations. Thus, resistivity of all metallic conductors increases with increase of temperature. At ordinary temperatures, the resistivity of most of the metals increases linearly with temperature.

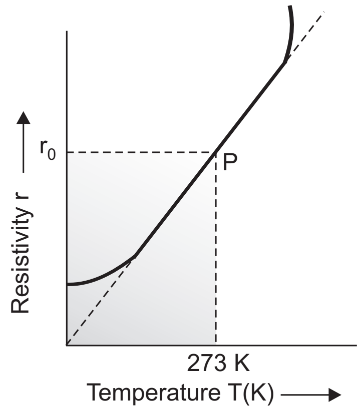


Fig. 2.13. Variation of resistivity with temperature

$$\rho_T = \rho_0[1 + \alpha(T - T_0)]$$

where ρ_0 is the resistivity at a reference temperature T_0, ρ_T is the resistivity at temperature T. The factor α is the temperature coefficient of resistivity.

At low temperatures, the temperature-dependence of resistivity is non-linear. At low temperatures, the resistivity increases as a higher power of temperature.

Graph (Fig. 2.14) shows the variation of resistivity of metallic conductors with temperature. The graph is a straight line over a limited range of temperature. The point P on the linear portion of the graph may be taken as the reference point. The temperature corresponding to this point is 273 K and resistivity is ρ_0.

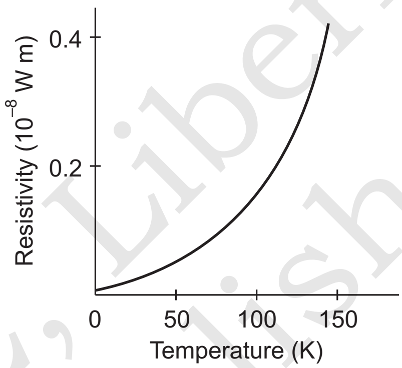


Fig. 2.14. Variation of resistivity of copper with temperature

Fig. 2.14 shows the resistivity of copper as a function of temperature T.

2.15 ELECTROMOTIVE FORCE OF A CELL

Electromotive force (emf) of a cell is the potential difference between the two terminals of the cell in an open circuit (when no current is drawn from the cell).

It is due to the emf of the cell that the cell drives the charge round the circuit. So, emf of a cell may also be defined as under :

Electromotive force (emf) of a cell is the energy supplied by the cell to drive a unit charge round the complete circuit. It is denoted by ε.

The SI unit of ε is joule/coulomb *i.e.*, volt.

The emf of a cell is said to be one volt if one joule of work is performed by the cell to drive one coulomb of charge round the circuit.

2.16 TERMINAL POTENTIAL DIFFERENCE OF A CELL

The terminal potential difference of a cell is the difference of potentials between the terminals of the cell when the cell is in closed circuit i.e., when current is drawn from the cell. It is denoted by V.

Note: Both emf and potential difference have the same units.

2.17 COMPARISON OF EMF AND POTENTIAL DIFFERENCE

1. The difference of potentials between the two terminals of a cell in an open circuit i.e., when no current is drawn from the cell is known as the electromotive force (emf) of the cell. On the other hand, the potential difference is the difference of potentials between any two points in a closed circuit.

2. The emf is independent of the resistance of the circuit. However, the potential difference between any two points of a circuit is proportional to the resistance between those two points.

3. The term 'emf ' is used only for the source of emf. The 'potential difference' is measured between any two points of the electric circuit.

4. The emf is greater than the potential difference between any two points of a circuit. However, when the cell is being charged, the potential difference between its two terminals is larger than the emf of the cell.

2.18 INTERNAL RESISTANCE OF A CELL

The opposition offered by the electrolyte of the cell to the flow of electric current through it is called the internal resistance of the cell. It is generally denoted by r. The value of internal resistance for a freshly prepared cell is low. However, as the cell is used more and more, its internal resistance goes on increasing.

The internal resistance of a cell depends upon the following factors :

(i) Larger the *separation between the electrodes* of the cell, more the length of the electrolyte through which current has to flow and consequently a higher value of internal resistance.

(ii) Greater the *conductivity* of the electrolyte, lesser is the internal resistance of the cell. So, the internal resistance depends upon the nature of the electrolyte.

(iii) The internal resistance of a cell is inversely proportional to the common area of the electrodes dipping in the electrolyte.

(iv) The internal resistance of a cell also depends upon the nature of the electrodes.

(v) The internal resistance of a cell increases with the decrease in temperature of the electrolyte.

2.19 CIRCUIT EQUATION

Consider a cell of emf ε and internal resistance r connected to an external resistance R as shown in Fig. 2.15. Let a constant current I flow through the circuit.

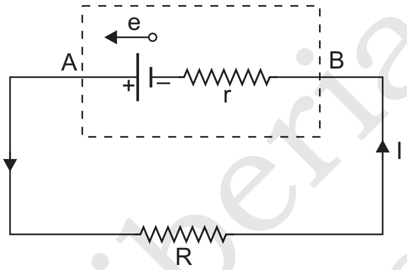


Fig. 2.15

Using definition of emf,

ε = Work done by the cell in carrying a unit charge along the closed circuit

= Work done in carrying a unit charge from A to B against the external resistance R + Work done in carrying a unit charge from B to A against the internal resistance r

$\varepsilon = V + V'$, where $V = IR$ and $V' = Ir$

$\therefore \quad \varepsilon = IR + Ir = I(R + r)$

or $$\boxed{I = \frac{\varepsilon}{R + r}}$$

This relation is called circuit equation.

Example 10: *Two cells ε_1 and ε_2 in the given circuit diagram have an emf of 5 V and 9 V and internal resistance of 0.3 Ω and 1.2 Ω respectively.*

Calculate the value of current flowing through the resistance of 3 Ω.

Solution: Net emf = 9 V – 5 V = 4 V

Total resistance $= 0.3 + 1.2 + 4.5 + \frac{6 \times 3}{6 + 3} = 8\ \Omega$

Current through the circuit, $I = \frac{4}{8} = 0.5$ A

Current through the 3 Ω resistance

$$= \frac{6 \times 0.5}{6 + 3} = \frac{1}{3}\ \text{A} = \mathbf{0.33\ A}$$

Example 11: *Three identical cells, each of emf 4 V and internal resistance r, are connected in series to a 6 Ω resistor. If the current flowing in the circuit is 1.5 A, calculate (a) internal resistance of each cell and (b) the terminal voltage across each cell.*

Solution: (*a*) Total emf = 3 × 4 V = 12 V

Total resistance = 6 + 3*r*

Using circuit equation,

$$1.5 = \frac{12}{6+3r} \quad \text{or } r = \frac{2}{3}\,\Omega = \mathbf{0.67}\ \Omega$$

$$(b)\ V = \varepsilon - Ir = \left(4 - 1.5 \times \frac{2}{3}\right) \text{volt} = \mathbf{3\ V}$$

2.20 KIRCHHOFF'S LAW

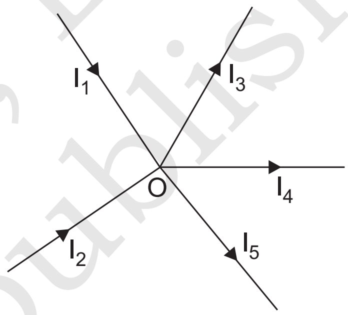


Fig. 2.16. Kirchhoff's current law

While discussing an electrical network, we shall proceed on the assumption that the emfs are constant and that the steady conditions have been reached in the network so that the currents are also constant. **The general problem is to calculate currents in terms of emfs and resistances.** There are two famous rules to solve this kind of problem. These rules were first developed by Gustav Robert Kirchhoff in the year 1842 and are known after him as Kirchhoff 's rules or Kirchhoff 's laws. **These laws are simply the expressions of conservation of electric charge and of energy**. These laws may be stated as follows :

1. Kirchhoff's first law or Kirchhoff 's Current Law or Junction Rule

It is stated as follows :

"***In any electrical network, the algebraic sum of currents meeting at a junction is always zero.***"

$$\Sigma I = 0$$

When applying this law, we adopt the following **sign convention** :

"*The currents directed towards the junction are taken as positive while those directed away from the junction are taken as negative.*"

Consider a number of conductors meeting at junction O and carrying currents I_1, I_2, I_3, I_4 and I_5. Applying Kirchhoff's current law to the network shown in Fig. 2.16, we get

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0 \quad \text{or} \quad I_1 + I_2 = I_3 + I_4 + I_5$$

This leads us to the following alternative way of stating the law :

"***The sum of the currents flowing towards a junction is equal to the sum of currents leaving the junction.***"

The charges cannot accumulate at a junction. The number of charges that arrive at a junction in a certain time must leave the junction in the same time. This is in accordance with the conservation of charge which is the basis of Kirchhoff 's current law.

Problem. Apply Kirchhoff's first law to the junctions A, B and C of the electrical network shown in Fig. 2.16.

Solution. For junction A : $I_6 - I_1 - I_2 = 0$

For junction B : $I_1 - I_3 - I_5 = 0$

For junction C : $I_2 + I_5 - I_4 = 0$

2. Kirchhoff's Second Law or Kirchhoff 's Voltage Law or Loop Rule

It is stated as follows :

"***The algebraic sum of all the potential drops and emfs along any closed path in a network is zero.***"

Kirchhoff's second law expresses the conservation of energy *i.e.,* the net change in the energy of a charge after the charge completes a closed path must be zero. When applying this law, we adopt the following **sign convention.**

In applying the second law, we have to keep in mind that ***the potential falls along the direction of current in a current path. It rises along a path from negative terminal to the positive terminal. While the 'potential fall' is taken as negative, the 'potential rise' is taken as positive.***

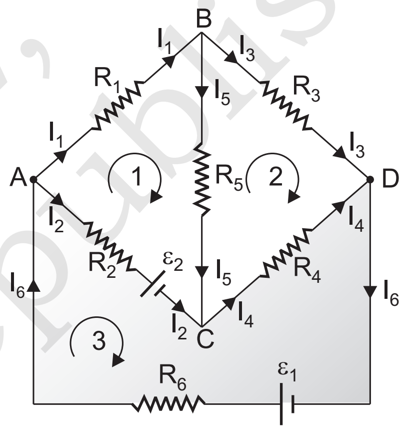


Fig. 2.17. Kirchhoff's voltage law

The product of current and resistance will be taken as negative when we traverse in the direction of the conventional current.

The emf is taken as negative when we traverse from positive to the negative terminal of the cell through the electrolyte. The emf will naturally be taken as positive when we traverse from negative to the positive terminal of the cell.

Keeping in mind this sign convention, let us now apply Kirchhoff's second law to the paths marked 1, 2 and 3 in Fig. 2.17.

For path 1, $- R_1I_1 - R_5I_5 + R_2I_2 + \varepsilon_2 = 0$

For path 2, $- R_3I_3 + R_4I_4 + R_5I_5 = 0$

For path 3, $- R_2I_2 - R_4I_4 - R_6I_6 + \varepsilon_1 - \varepsilon_2 = 0$

or $\varepsilon_1 - \varepsilon_2 = R_2I_2 + R_4I_4 + R_6I_6$

In general, $\Sigma \varepsilon = \Sigma IR$

A practical rule to follow in finding the currents in a network having n junctions is to apply the first law to $(n - 1)$ junctions only because once the law is satisfied for $(n - 1)$ junctions, it is automatically satisfied for the remaining junction. Kirchhoff's second law must be applied to as many closed paths as required in order for each conductor to be part of a path at least once.

The following guidelines will help with the problem of signs while applying Kirchhoff's second law :

(i) Choose any closed loop in the network and designate a direction (clockwise or counter clockwise) to traverse the loop.

(ii) Go around the loop in the designated direction, adding emfs and potential differences. An emf is counted as 'positive' when it is traversed from negative to positive and 'negative' when from positive to negative. An IR term is counted as negative if the resistor is traversed in the same direction as the assumed current, positive if in the opposite direction.

2.21 COMPARISON OF KIRCHHOFF'S FIRST AND SECOND LAWS

1. While first law is in accordance with the law of conservation of charge, the second law is in accordance with the law of conservation of energy.
2. The first law is applicable to both open and closed circuits. The second law is applicable to closed circuit.
3. According to first law, $\Sigma I = 0$. According to second law, $\Sigma \varepsilon = \Sigma IR$.

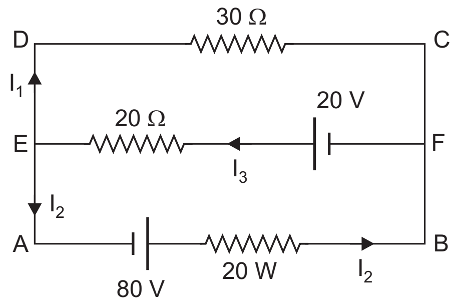


Fig. 2.18

Example 14: *Use Kirchhoff's rules to determine the value of current I_1 flowing in the circuit shown in the figure.*

Solution: Using Kirchhoff's first law at junction E,

$$I_3 = I_1 + I_2 \quad \ldots(i)$$

In loop ABCDA, using Kirchhoff's second law

$$80 - 20I_2 + 30\,I_1 = 0 \quad \text{or} \quad 2I_2 - 3I_1 = 8 \quad \ldots(ii)$$

In loop ABFEA, we get

$$80 - 20\,I_2 + 20 - 20\,I_3 = 0$$

or $$I_2 + I_3 = 5 \qquad \ldots(iii)$$

Putting the value of I_3 in (*iii*), we have

$$I_2 + (I_1 + I_2) = 5$$

or $$2I_2 + I_1 = 5 \qquad \ldots(iv)$$

Solving equations (*ii*) and (*iv*), we get

$$I_1 = -\frac{3}{4}\,\text{A} = \mathbf{-0.75\ A}$$

The negative sign indicates that the direction of current is opposite to that shown in the circuit diagram.

2.22 WHEATSTONE BRIDGE (Simple Application of Kirchhoff's Laws)

It is an arrangement of four resistances used for measuring one of them in terms of the other three. It was devised by Sir Charles F. Wheatstone, a British physicist in 1833. It is in his honour that the arrangement is known as Wheatstone bridge.

Fig. 2.19 represents Wheatstone's bridge circuit where P, Q, R and S are connected to form a mesh. A battery of emf ε is connected between the junctions A and C through a key K_B called the battery key. A galvanometer of resistance G is connected between the terminals B and D through a key K_G called galvanometer key. *It is always the battery key which is closed first and then the galvanometer key.*

The currents through the various branches are indicated in Fig. 2.19. *In order to reduce the number of unknowns at the outset, Kirchhoff's first rule is used at every junction.*

Applying Kirchhoff's voltage law to the mesh ABDA, we get

$$-I_1P - I_gG + (I - I_1)R = 0 \qquad \ldots(1)$$

Again, applying Kirchhoff's voltage law to the mesh BCDB, we get

$$-(I_1 - I_g)Q + (I - I_1 + I_g)S + I_gG = 0 \qquad \ldots(2)$$

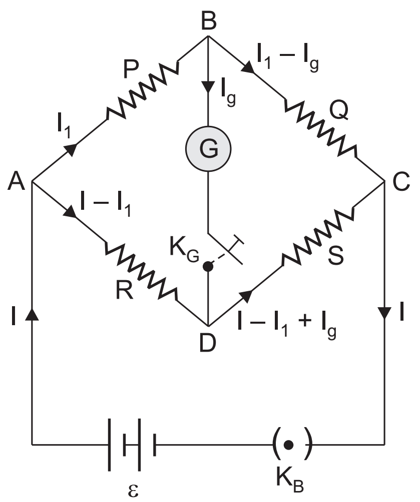


Fig. 2.19. Wheatstone bridge

The resistances P, Q, R and S are so adjusted that the galvanometer gives zero deflection. This would be possible if B and D are at the same potential and therefore, no current would flow through the galvanometer *i.e.,* $I_g = 0$. In this situation, the Wheatstone bridge is said to be **balanced.**

Substituting $I_g = 0$ in equations (1) and (2), we get

$$-I_1P + (I - I_1)R = 0 \quad \text{...(3)}$$

and

$$-I_1Q + (I - I_1)S = 0 \quad \text{...(4)}$$

Rewriting (3) and (4), we have

$$I_1P = (I - I_1)R \quad \text{and} \quad I_1Q = (I - I_1)S$$

Dividing, $$\frac{P}{Q} = \frac{R}{S}$$

This is the condition for the Wheatstone bridge to be balanced. Clearly, if three resistances are known, the fourth one can be calculated.

Example 16: *The galvanometer, in each of the two given circuits, does not show any deflection. Find the ratio of the resistors R_1 and R_2, used in these two circuits.*

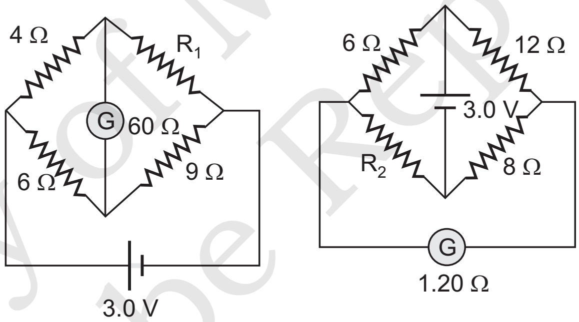


Fig. 2.20

Solution: For the first circuit, we have (from the Wheatstone bridge balance condition),

$$\frac{4}{R_1} = \frac{6}{9} \text{ or } R_1 = 6\ \Omega$$

For the second circuit, the interchange of the positions of the battery and the galvanometer does not change the Wheatstone bridge balance condition.

$$\therefore \quad \frac{6}{12} = \frac{R_2}{8} \text{ or } R_2 = 4\ \Omega$$

Now, $$\frac{R_1}{R_2} = \frac{6}{4} = \mathbf{\frac{3}{2}}$$

2.23 THERMAL EFFECT OF CURRENT AND JOULE'S LAW

The heating of a conductor by the flow of an electric current through it is called Joule heating. It is an irreversible process. If the direction of current in a resistor is reversed, the resistor would not be cooled. It would again show heating effect.

Joule's law of heating may be stated as follows :

The heat produced in a conductor due to flow of current in it is proportional to square of current, resistance of conductor and the time for which current flows.

In 1841, Joule stated that when a current I is made to flow through a passive or ohmic resistance R for time t, heat Q is produced such that

$$Q = I^2Rt$$

This equation is called **Joule's law of heating**. It is clear that the amount of heat developed in a passive resistance by the passage of steady electric current through it is proportional to

(i) the square of the electric current **(ii)** the resistance of conductor **(iii)** the time for which the current flows.

2.24 ELECTRICAL ENERGY AND POWER

(*i*) ***Electrical energy** is the total work done by an electric current in a given time. It is equal to the total energy consumed in an electric circuit in a given time.*

Consider a conductor AB through which a steady current I is flowing from A to B [Fig. 2.21]. Let V_A and V_B be the potentials at the ends A and B respectively of the conductor.

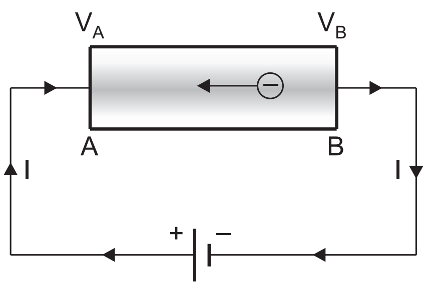


Fig. 2.21

Since current flows from A to B, ∴ $V_A > V_B$.

The electrons move from B to A. They enter B at potential V_B and leave A at potential V_A.

The electric potential at a point is the potential energy per unit charge at that point.

Potential energy of an electron at B = $(-e)\ V_B$

Potential energy of an electron at A = $(-e)\ V_A$

Since V_A is greater than V_B, therefore, the electrons entering B possess more potential energy than those leaving A. But their kinetic energies are the same because their drift velocities are equal. So, the electrons lose potential energy while passing from B to A.

Let n be the number of electrons entering B and leaving A in time dt. If e be the charge on electron, then charge flowing through AB in time dt, $dq = ne$.

Current, $$I = \frac{dq}{dt} = \frac{ne}{dt} \quad \text{or} \quad n = \frac{Idt}{e}$$

Loss in potential energy of electrons in time dt,

$$dW = \frac{Idt}{e}[(-e)\ V_B - (-e)\ V_A] = Idt\ (V_A - V_B) = Idt\ V_{AB}$$

where V_{AB} is the potential difference between A and B.

This energy appears as heat energy. In general, if a current of I ampere flows through a wire of resistance R ohm for t second when the potential difference between the ends of the wire is V volt, then the electrical energy is given by :

$$W = VIt \text{ joule}$$

But $$V = IR \text{ (Ohm's law)}$$

$\therefore$ $$W = I^2Rt \text{ joule}$$

1 calorie of heat is equivalent to 4.2 joule work. So, the heat Q produced in the wire is given by

$$Q = \frac{W}{4.2} = \frac{VIt}{4.2} \text{ cal} = \frac{I^2Rt}{4.2} \text{ cal} = \frac{V^2t}{4.2\ R} \text{ cal} = 0.238 I^2Rt \text{ cal}$$

$$\approx 0.24\ I^2Rt \text{ cal}$$

which is Joule's law of heating.

(*ii*) ***Electric power*** *is the rate at which work is done by an electric current.*

The electric power of an appliance is the rate of consumption of electric energy.

Energy consumed in time t, W = VIt joule

Electric power, $$P = \frac{W}{t} = \frac{VIt}{t} \text{ joule s}^{-1} \text{ or W}$$

or $\boxed{P = VI}$ or $\boxed{P = I^2R}$ or $\boxed{P = \dfrac{V^2}{R}}$

(*iii*) **Units of power.** If V is measured in volt and I in ampere, then P is measured in watt.

$$1 \text{ watt} = 1 \text{ volt} \times 1 \text{ ampere}$$

The power expended is said to be one watt when a current of one ampere flows under a potential difference of one volt.

A bigger unit of power is kilowatt. 1 kW = 1000 W

$$\text{Power in kilowatt} = \frac{\text{V in volt} \times \text{I in ampere}}{1000}$$

The power of an electrical appliance is generally expressed in kilowatt.

The engineering unit of power is 'horse power (hp)'. 1 hp = 746 W

(*iv*) **Commercial unit of electrical energy.** The commercial unit of electric energy is kilowatt hour or Board of Trade Unit (**kWh or B.O.T.U.**). *It is defined as the amount of work done when a power of one kilowatt is consumed for one hour.*

$$\begin{aligned} \mathbf{1\ kWh} &= 1 \text{ kW} \times 1 \text{ hour} \\ &= 1000 \text{ W} \times 3600 \text{ s} = 1000 \text{ J s}^{-1} \times 3600 \text{ s} \\ &= \mathbf{3.6 \times 10^6\ J = 3.6 \times 10^{13}\ erg} \qquad [\because \quad 1 \text{ J} = 10^7 \text{ erg}] \end{aligned}$$

Note. The watt-hour-meter placed on the premises of every consumer records the electrical energy consumed.

REVIEW EXERICSE

A. MULTIPLE CHOICE QUESTIONS (MCQs)

1. The mean free path of electrons in a metal is 4×10^{-8} m. The electric field which can give on an average 2 eV energy to an electron in the metal will be in unit of V m^{-1}

(*a*) 8×10^7 (*b*) 5×10^{-11} (*c*) 8×10^{-11} (*d*) 5×10^7

2. A quantity *X* is given by $\varepsilon_0 L(\Delta V/\Delta t)$ where ε_0 is permittivity of free space, *L* is length, ΔV is potential difference and Dt is time interval. The quantity *X* is same as

(*a*) resistance (*b*) charge (*c*) voltage (*d*) current

3. The temperature coefficient of resistance of the material of a wire is 0.001 per °C. Its resistance at 300 K is 1 ohm. At what temperature will the resistance of the wire be 2 ohm ?

(*a*) 781 K (*b*) 1027 K (*c*) 1054 K (*d*) 1327 K

4. Dimensions of resistance in an electrical circuit, in terms of dimension of mass *M,* of length *L*, of time *T* and of current *I*, would be :

(*a*) ML^2T^{-2} (*b*) $ML^2T^{-1}I^{-1}$ (*c*) $ML^2T^{-3}I^{-2}$ (*d*) $ML^2T^{-3}I^{-1}$

5. Three resistors 1 W, 2 W and 3 W are connected to form a triangle. Across 3 W resistor, a 3 V battery is connected. The current through 3 W resistor is

(*a*) 0.75 A (*b*) 1 A (*c*) 2 A (*d*) 1.5 A

6. The density of copper is 9×10^3 kg m^{-3} and its atomic mass is 63.5 u. Each copper atom provides one free electron. Estimate the number of free electrons per cubic metre in copper.

(*a*) 10^{19} (*b*) 10^{23} (*c*) 10^{25} (*d*) 10^{29}

7. A colour coded carbon resistor has the colours orange, blue, green and silver. Its resistance value and tolerance percentage respectively are

(*a*) 36×10^5 W and 10% (*b*) 36×10^4 W and 5%

(*c*) 63×10^5 W and 10% (*d*) 35×10^6 W and 5%

8. The dimensions of 'resistance' are same as those of ...... where h is the Planck's constant, *e* is the charge.

(*a*) $\frac{h^2}{e^2}$ (*b*) $\frac{h^2}{e}$ (*c*) $\frac{h}{e^2}$ (*d*) $\frac{h}{e}$

9. If a wire is stretched to make it 0.1% longer, its resistance will

(*a*) increase by 0.05% (*b*) increase by 0.2%

(*c*) decrease by 0.2% (*d*) decrease by 0.05%

10. If 400 W of resistance is made by adding four 100 W resistance of tolerance 5%, then the tolerance of the combination is

(*a*) 20% (*b*) 5% (*c*) 10% (*d*) 15%

11. The resistance of a 10 m long wire is 10 W. Its length is increased by 25% by stretching the wire uniformly. Then the resistance of the wire will be

(*a*) 12.5 W (*b*) 14.5 W (*c*) 15.6 W (*d*) 16.6 W

(*e*) 18.6 W

12. Two identical conductors maintained at same temperature are given potential differences in the ratio 1 : 2. Then the ratio of their drift velocities is

(*a*) 1 : 2 (*b*) 3 : 2 (*c*) 1 : 1 (*d*) $1 : 2^{1/2}$

13. A resistor has the following colour code, sequentially from the left: Black Brown Orange Red and Black.
What is the resistance of the resistor ?
(*a*) 13 Ohm (*b*) 1300 Ohm
(*c*) 130 Ohm (*d*) 13000 Ohm.

14. A coil has resistance 25.00 ohm and 25.17 ohm at 20°C and 35°C respectively. What is the temperature coefficient of resistance ?
(*a*) 4.545×10^{-4}/°C (*b*) 4.545×10^{-3}/°C
(*c*) 4.545×10^{-2}/°C (*d*) 4.545×10^{-5}/°C.

15. The number of electrons per second flowing through any cross-section of the wire carrying current of 1 ampere is
(*a*) 3.12×10^{16} (*b*) 1.6×10^{18} (*c*) 6.25×10^{16} (*d*) 3.12×10^{18}
(*e*) 6.25×10^{18}

B. FILL IN THE BLANKS

1. Two batteries of emfs 2 V and 1 V of internal resistances 1 W and 2 W respectively are connected in parallel. The effective emf of the combination is ....
2. A battery of emf 8 V with internal resistance 0.5 W is being charged by a 120 V dc supply using a series resistance of 15.5 W. The terminal voltage of the battery is ....
3. The resistances in the four arms of a Wheatstone network in cyclic order are 5 Ω, 2 Ω, 6 Ω and 15 Ω. If a current of 2.8 A enters the junction of 5 Ω and 15 Ω, then the current through 2 Ω resistor is ....
4. A galvanometer connected with an unknown resistor and two identical cells in series each of emf 2 V shows a current of 1 A. If the cells are connected in parallel, it shows 0.8 A. Then the internal resistance of the cell is ....
5. The percentage error in measuring resistance with a metre bridge can be minimised by adjusting the balancing point close to ....

C. VERY SHORT ANSWER TYPE QUESTIONS

1. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery ?
2. A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor ? What is the terminal voltage of the battery when the circuit is closed ?

3. At room temperature (27.0°C), the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is 1.70×10^{-4} °C^{-1}.

4. A silver wire has a resistance of 2.1 Ω at 27.5°C, and a resistance of 2.7 Ω at 100°C. Determine the temperature coefficient of resistance of silver.

5. A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging ? What is the purpose of having a series resistor in the charging circuit ?

D. SHORT ANSWER TYPE QUESTIONS

1. (*a*) Three resistors 1 Ω , 2 Ω and 3 Ω are combined in series. What is the total resistance of the combination ?

(*b*) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.

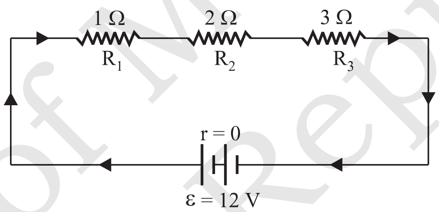


Fig. 2.22

2. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell ?

3. The number density of free electrons in a copper conductor is 8.5×10^{28} m^{-3}. How long does an electron take to drift from one end of a wire 3.0 m long to its other end ? The area of cross-section of the wire is 2.0×10^{-6} m^2 and it is carrying a current of 3.0 A.

4. A negligibly small current is passed through a wire of length 15 m and uniform cross-section 6.0×10^{-7} m^2, and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment ?

5. Two wires of equal length, one of aluminium and the other of copper, have the same resistance. Which of the two wires is lighter ? Hence explain why aluminium wires are preferred for over-head power cables.

(ρ_{Al} = 2.63 × 10^{-8} Ω m, ρ_{Cu} = 1.72 × 10^{-8} Ω m, Relative density of Al = 2.7, of Cu = 8.9)

E. LONG ANSWER TYPE QUESTIONS

1. Two batteries, each of emf E and internal resistance *r*, are connected in parallel. The current from this combination is sent through an external resistance R. For what value of R, maximum power will be obtained? What will be this maximum power?

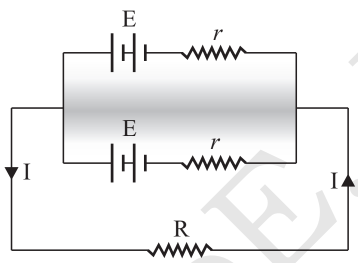


Fig. 2.23

2. Three equal resis-tances, each of R ohm, are connected as shown in Fig. 2.24. A battery of emf 2 V and internal resistance 0.1 Ω is connected across the circuit. Calculate the value of R for which the heat generated in the circuit is maximum?

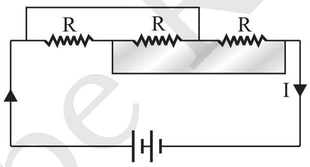


Fig. 2.24

3. A heater is designed to operate with a power of 1000 W in a 100 volt line [Fig. 2.25]. It is connected to two resistances of 10 Ω and R Ω. If the heater is now operating with a power of 62.5 W, calculate the value of R.

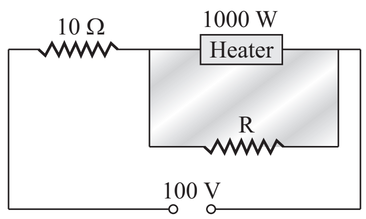


Fig. 2.25

4. Determine the current in each branch of the network shown in figure.

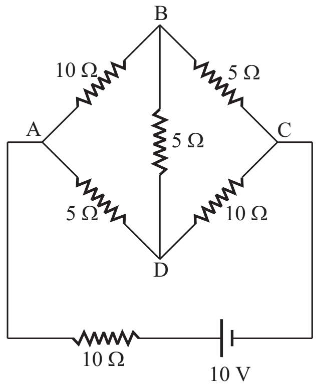


Fig. 2.26

5. (*a*) In a metre bridge shown the figure, the balance point is found to be at 39.5 cm from the end A, when the resistor Y is of 12.5 Ω. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or metre bridge made of thick copper strips ?

(*b*) Determine the balance point of the bridge if X and Y are interchanged.

(*c*) What happens if the galvanometer and cell are interchanged at the balance point of the bridge ? Would the galvano-meter show any current ?

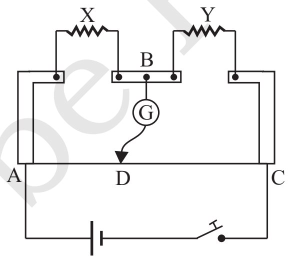


Fig. 2.27

PERIOD-3

TOPIC 3

Magnetism and Electro-Magnetism

3.1 BAR MAGNET AND ITS PROPERTIES

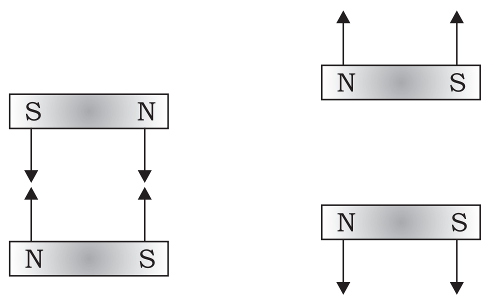
A piece of iron can be treated in such a way as to produce a magnet. The magnetic properties of a magnet can also be destroyed in several ways, one of which is by heating it. Ordinarily, the ends of the magnet are the centres of this attraction. The two centres of attraction in any magnet are called its **poles.** After a magnet is suspended so that it is free to rotate about a vertical axis and it has lined up along a meridian, the pole in the northern end of the magnet is called a north pole and the pole in the southern end is called a south pole. Forces are exerted between the poles of two adjacent magnets such that unlike poles attract each other while like poles repel each other.

The region around a magnet in which it exerts forces on other magnets and on objects made of iron is called a ***magnetic field.***

The main **properties** of a bar magnet can be summed up as follows :

1. Directive property. When a magnet is suspended freely, it aligns itself with one end pointing towards north of earth and other towards south of earth.

2. Force between poles. Unlike poles attract and like poles repel (Fig. 3.1). The force of attraction or repulsion acts along the line joining the two poles and is directly proportional to the product of the pole strengths and inversely proportional to the square of the distance between them.



(a) Unlike poles attract (b) Like poles repel

Fig. 3.1

3. Attractive property. A magnet attracts magnetic materials such as iron, steel, cobalt, nickel etc. This attraction is maximum at the poles. *It may be noted that the poles are situated near the ends of the magnet and not exactly at the ends.*

4. Isolated magnetic pole does not exist *i.e.*, magnetic poles always exist in pairs. The similarity in the behaviour of electric charges and poles seems to suggest the possibility of 'magnetic charges'—N-type and S-type. But, so far, every attempt at finding single magnetic charges or to isolate the poles of a magnet from one another have failed. If we break a long bar magnet into two (or even more) pieces, then we shall observe that each piece is complete magnet in itself (Fig. 3.2).

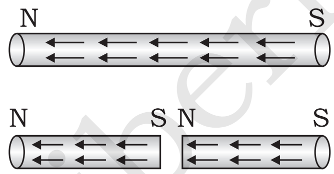


Fig. 3.2. Breaking of a magnet.

5. A magnet can induce magnetism in substances like soft iron, cobalt, nickel and various ferrous alloys. When an iron, nickel or cobalt rod is placed near a bar magnet, the bar magnet induces magnetism in the rod.

3.2 REPULSION—SUREST TEST OF MAGNETISATION

An iron rod is attracted towards a magnet. The opposite poles also attract each other. So, attraction is not a sure test of magnetisation. On the other hand, if there is repulsion between a magnet and a given rod, we can be sure that the given rod is magnetised.

3.3 ATOMIC THEORY OF MAGNETISM

Both the orbital and spin motions of electrons give rise to tiny circular currents. These tiny current loops behave like small magnets. These small magnets are called elementary or atomic magnets. When the material is not magnetised, these magnets form closed chains thereby annulling each other's effect (Fig. 3.3).

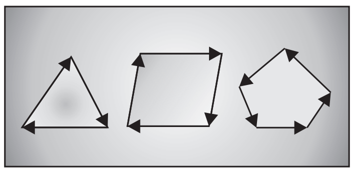


Fig. 3.3. Closed chains of elementary magnets.

When the material is magnetised, the elementary magnets are aligned nearly in the same direction (Fig. 3.4).

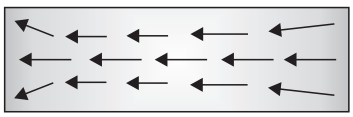


Fig. 3.4. Alignment of elementary magnets.

The atomic theory of magnetism explains the following facts :

(i) Single poles cannot exist. Poles always exist in pairs.

(ii) The magnetic poles are of equal strength.

(iii) When a magnet is heated, the thermal energy of the elementary magnets increases. These again form closed chains and magnetism is lost.

3.4 THE MAGNETIC FIELD LINES

We begin our study by examining iron filings sprinkled on a sheet of glass placed over a short bar magnet. The arrangement of iron filings is shown in Fig. 3.5.

The pattern of iron filings suggests that the magnet has two poles similar to the positive and negative charges of an electric dipole. One pole is designated the north pole and the other, the south pole. When suspended freely, these poles point approximately towards the geographic north and south poles, respectively. A similar pattern of iron filings is observed around a current-carrying solenoid.

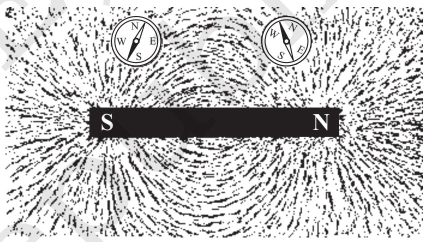


Fig. 3.5. The arrangement of iron filings surrounding a bar magnet. The pattern mimics magnetic field lines. They suggest that the bar magnet is a magnetic dipole.

The pattern of iron filings permits us to plot the magnetic field lines. This is shown both for the bar magnet and the current-carrying solenoid in Fig. 3.6. The magnetic field lines are a visual and intuitive realisation of the 'unseen' magnetic field.

Some **important properties** of magnetic field lines are as under :

(i) Outside the body of the magnet, the magnetic field lines are **directed from north pole to south pole.**

(ii) Magnetic field lines have a **tendency to contract** longitudinally. This explains attraction between unlike magnetic poles.

(iii) Magnetic field lines have a tendency to exert **lateral pressure.** This explains repulsion between like magnetic poles.

(iv) The magnetic field lines of a magnet (or a solenoid) **form continuous closed loops.**

(v) The **tangent to the field line** at a given point represents the direction of the net magnetic field $\vec{B}$ at that point.

(vi) The **larger the number of field lines** crossing per unit normal area, the larger is the magnitude of the magnetic field $\vec{B}$. In Fig. 3.6(*a*), $\vec{B}$ is larger around region II than in region I.

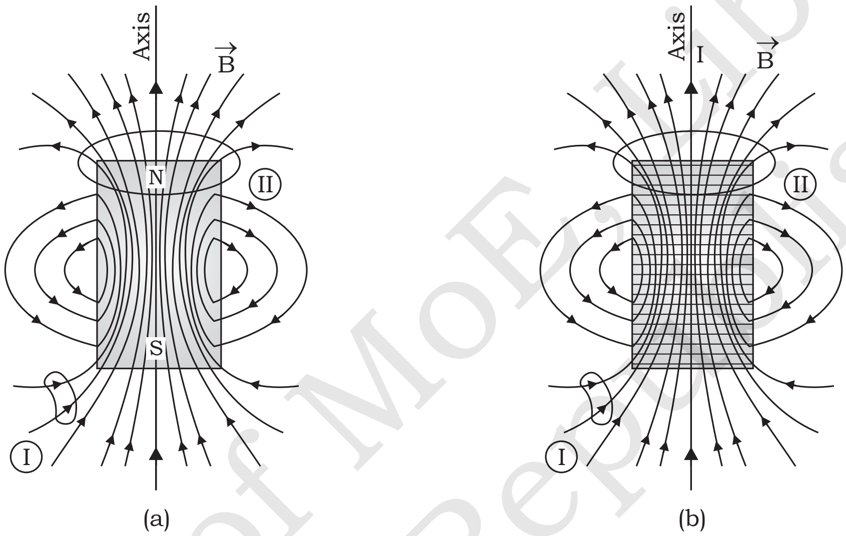


Fig. 3.6. The field lines of (*a*) a bar magnet, (*b*) a current-carrying finite solenoid. At large distances, the field lines are very similar.

(vii) The magnetic field lines **do not intersect.** This is so since the direction of the magnetic field would not be unique at the point of intersection.

One can plot the magnetic field lines in a variety of ways. One way is to place a small magnetic compass needle at various positions and note its orientation. This gives us an idea of the magnetic field direction at various points in space.

3.5 REPRESENTATION OF UNIFORM MAGNETIC FIELD

*A **magnetic field** is said to be uniform over a region if its magnetic field induction $\vec{B}$ has the same magnitude and direction at all points in the region.*

Example. Earth's magnetic field may be regarded as uniform over a small region.

If $\vec{B}$ varies in magnitude or in direction or both, then the magnetic field is non-uniform.

A uniform magnetic field in the plane of paper is represented by a set of equal, equidistant and parallel lines pointing in the same direction as shown in Fig. 3.7 (*a*).

A uniform magnetic field perpendicular to the plane of paper and directed outwards is represented by a group of equidistant tiny dots as shown in Fig. 3.7 (*b*).

A uniform magnetic field perpendicular to the plane of paper and directed inwards is represented by a group of equidistant crosses as shown in Fig. 3.7 (*c*).

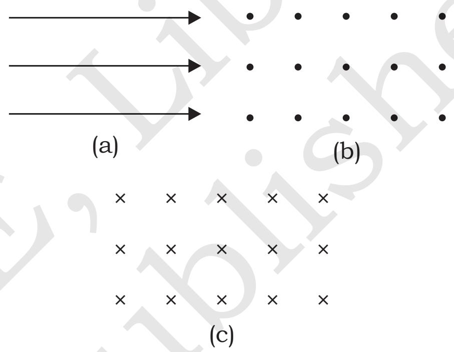


Fig. 3.7. Representation of uniform magnetic field.

3.6 MAGNETIC DIPOLE AND MAGNETIC DIPOLE MOMENT

A pair of magnetic poles of equal and opposite strengths separated by a finite distance is called a **magnetic dipole.**

Examples. Bar magnet, compass needle and current loop.

Magnetic length of a dipole is the distance between the two poles. It is a vector directed from S-pole to the N-pole. It is represented by $\vec{2l}$.

The magnetic dipole moment $\vec{m}$ *(also denoted by* $\vec{p_m}$ *) is a vector directed from south pole to the north pole along the axis of the dipole. The magnitude of the dipole moment is the product of the pole strength* q_m *and the separation 2l between the poles.*

$$\boxed{m = q_m \times 2l} \quad \text{In vector notation, } \vec{m} = q_m(\vec{2l})$$

The SI unit of m is A m^2 or J T^{-1}.

3.7 MAGNETISATION AND DEMAGNETISATION

A popular theory of magnetism considers the molecular alignment of the material. This is known as Weber's theory. This theory assumes that all magnetic substances are composed of tiny molecular magnets.

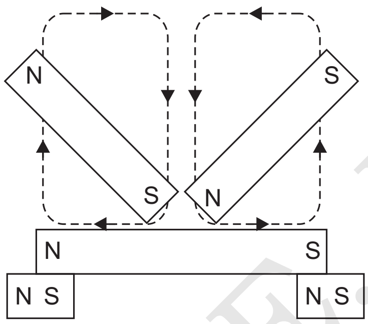


Fig. 3.8. Current loop as a magnetic dipole.

Any unmagnetized material has the magnetic forces of its molecular magnets neutralized by adjacent molecular magnets, thereby eliminating any magnetic effect. A magnetized material will have most of its molecular magnets lined up so that the north pole of each molecule points in one direction, and the south pole faces the opposite direction. A material with its molecules thus aligned will then have one effective north pole, and one effective south pole.

An illustration of Weber's Theory is shown in Fig. 3.8, where a steel bar is magnetized by stroking. When a steel bar is stroked several times in the same direction by a magnet, the magnetic force from the north pole of the magnet causes the molecules to align themselves.

Ways of making magnets

1. **'Stroke' method:** A piece of magnetic material can be truned into a magnet if it is stroked by a magnet. As the magnet moves along the magnetic material, it causes the magnetic dipoles in the magnetic material to become aligned in one direction and give rise to a magnetic field.
2. **Electrical method using a direct current:** When a large direct current is passed through the solenoid, the unmagnetised steel bar will become magnetized after a while. This is because when an electric current flows through the solenoid, it produces a strong magnetic field which magnetizes the steel bar.

The poles of the magnet can be determined by a simple method known as Right-hand grip rule.

Ways of damagnetizing magnets

1. **Heating:** Heating a piece of magnetized metal in a flame will cause demagnetization by destroying the long-range order of molecules within the magnet. By heating a magnet, each molecule is infused with energy. This forces it to move, pushing each molcule out of order within the magnet and leaving the piece of metal with very little or no magnetization.
2. **Hammering:** When a magnet is hammered or dropped, the vibrations caused by the impact on the magnet randomize the magnetic molecules within the magnet, forcing them out of order and destroying the long-range order of the unit magnet.
3. **Alternating Current (AC) field:** Using an AC current produces a magnetic field which can be moved and reduced to demagnetize materials. The field created by the AC current drags the magnetic molecules of the magnet in different directions. When the AC current is altered or reduced, the molecules within the magnet do not all return to previous positions, causing randomization of the molecules and reducing the force of the magnet.

3.8 ELECTROMAGNETIC INDUCTION

The phenomenon of generation of an emf in a coil due to change in magnetic flux linked with it is known as *electromagnetic induction*. The emf so produced is called induced emf. If the coil is closed, a current flows in the coil. This current is called induced current.

3.9 MAGNETIC FLUX

(*i*) *The magnetic flux linked with a surface held in a magnetic field is defined as the total number of magnetic field lines crossing the surface normally.*

Magnetic flux is a scalar quantity and is denoted by ϕ_B.

Magnetic flux through a plane of area A placed in a uniform magnetic field $\vec{B}$ is the dot product of magnetic field vector $\vec{B}$ and area vector $\vec{A}$.

$$\therefore \quad \phi_B = \vec{B} \cdot \vec{A} = BA \cos\theta \qquad ...(1)$$

where θ is the angle between $\vec{B}$ and $\vec{A}$.

Magnetic flux is measured as the product of the component of the magnetic field normal to the surface and the surface area.

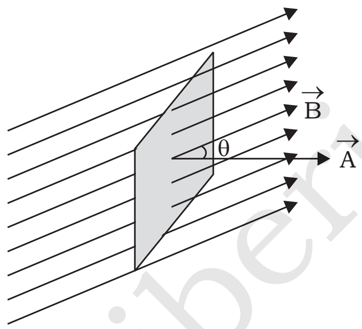


Fig. 3.9. A plane of surface area $\vec{A}$ placed in a uniform magnetic field $\vec{B}$.

The magnetic flux linked with a surface area ΔA held inside a magnetic field B is given by :

$$\Delta\phi_B = B_n \Delta A$$

Here, B_n is the component of magnetic field B along the normal to the surface area ΔA.

$$B_n = B \cos\theta$$

where θ is the angle which the normal to the surface area makes with the direction of the magnetic field.

$$\therefore \qquad \Delta\phi_B = B\,\Delta A \cos\theta \quad \text{or} \quad \Delta\phi_B = \vec{B} \cdot \Delta\vec{A}$$

If $\hat{n}$ is a unit vector along the normal to the plane surface area, then

$$\Delta\phi_B = \vec{B} \cdot (\hat{n}\,\Delta A)$$

Here, $\vec{B} \cdot \hat{n}$ (= B_n) represents the component of the magnetic field along the normal to the surface area.

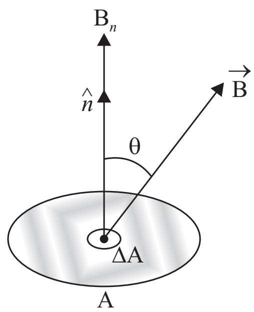


Fig. 3.10

(ii) If the magnetic field B under consideration is uniform over surface area A, then

$$\phi_B = B_n A \quad \text{and} \quad \phi_B = \vec{B} \cdot \vec{A}$$

(iii) If the magnetic field $\vec{B}$ is not uniform, then the surface is divided into a large number of area elements. Each area element should be small enough so that magnetic field through that element is taken as uniform. The total magnetic flux ϕ_B linked with complete area A is given by the algebraic sum of the magnetic fluxes linked with all elementary areas of the surface.

(*iv*) Positive magnetic flux

If the normal is drawn in the direction of the magnetic field, then the flux is taken as positive. In this case, θ is 0°. Even if θ is an acute angle, the flux is taken as positive.

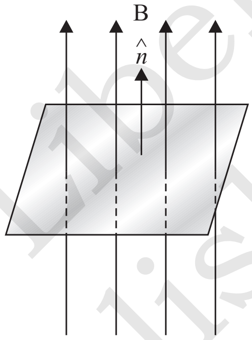


Fig. 3.11. Maximum magnetic flux

(*v*) Negative magnetic flux

If the normal is drawn opposite to the direction of the field, then θ = 180°. In this case, the magnetic flux is taken as negative. In fact, an obtuse value of angle θ makes cos θ negative and hence the magnetic flux is negative.

(*vi*) Maximum magnetic flux

When the uniform magnetic field is normal to the plane of the surface, then θ = 0°.

$$\therefore \qquad \phi_B = BA \cos 0°$$

$$\text{or} \qquad \phi_B = BA \qquad \text{(maximum value)}$$

So, when the magnetic field is normal to the surface, the magnetic flux through the surface will be maximum *i.e.,* maximum number of lines will pass normally through the given surface.

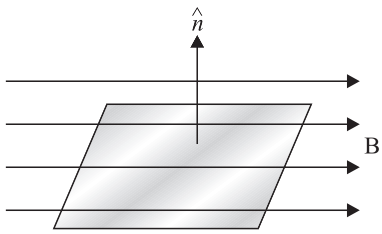


Fig. 3.12. Minimum magnetic flux

(*vii*) Minimum (zero) magnetic flux

When the magnetic field is parallel to the plane of the surface, then θ = 90°.

$$\therefore \qquad \phi_B = BA \cos 90° = 0$$

(*viii*) Units of magnetic flux

(*i*) The SI unit of magnetic flux is weber (Wb).

$$1 \text{ Wb} = 1 \text{ T} \times 1 \text{ m}^2$$

One weber is the magnetic flux linked with a surface area of 1 m^2 when held normally inside a uniform magnetic field of 1 tesla.

(*ii*) The *cgs* unit of magnetic flux is maxwell (Mx.)

$$1 \text{ Mx} = 1 \text{ G} \times 1 \text{ cm}^2$$

One maxwell is the magnetic flux linked with a surface area of 1 cm² when held normally inside a uniform magnetic field of 1 gauss.

A single magnetic line of force is known as a maxwell of flux.

(*ix*) Relation between units of magnetic flux

$$1 \text{ Wb} = 1 \text{ T} \times 1 \text{ m}^2 = 10^4 \text{ G} \times 10^4 \text{ cm}^2 = 10^8 \text{ Mx}$$

(*x*) Dimensional formula of magnetic flux

$$\phi_B = BA \cos\theta$$

Now, $F_m = Bqv$ or $B = \frac{F_m}{qv}$

$\therefore$ $\phi_B = \frac{F_m}{qv} A \cos\theta$

$$[\phi_B] = \frac{[MLT^{-2}][L^2]}{[AT][LT^{-1}]} = \frac{[ML^2T^{-2}]}{[A]} = [ML^2\, T^{-2}A^{-1}]$$

Electromagnetic induction: *It is the relative motion between the magnet and the coil that is responsible for generation (induction) of electric current in the coil.*

When magnetic flux changes through a coil, a current is induced in the coil. Quicker the relative motion between the magnet and the coil, greater is the rate of change of magnetic flux through the coil and larger is the current induced in it. This is the **elementary idea of electromagnetic induction.**

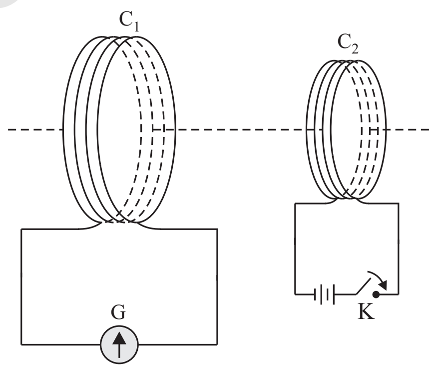


Fig. 3.13. Experimental set-up for Experiment No. 3.

Electromagnetic induction *is the process in which an emf is induced in a circuit placed in a magnetic field when the magnetic flux linked with the circuit changes.*

It is observed that the galvanometer shows a momentary deflection when the tapping key K is pressed. The pointer in the galvanometer returns to zero immediately. If the key is held pressed continuously, there is no deflection in the galvanometer. When the key is released, a momentory deflection is observed again, but in the opposite direction. It is also observed that the deflection increases dramatically when an iron rod is inserted into the coil along their axis.

3.10 FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

On the basis of the above experiments, Faraday formulated two laws of electromagnetic induction stated as follows :

First law. ***Whenever there is a change in the magnetic flux linked with a circuit, an emf and consequently a current is induced in the circuit. However, it lasts only so long as the magnetic flux is changing.***

Second law. ***The magnitude of the induced emf is directly proportional to the rate of change of magnetic flux linked with the circuit.***

It may be noted that Faraday's laws of electromagnetic induction tell us nothing about the direction of the induced emf and current. This direction is given by Lenz's law.

3.11 LENZ'S LAW

In 1834, German physicist Heinrich Friedrich Lenz [1804–1865] deduced a law, now known as Lenz's law. This law describes the polarity of the induced emf very clearly. The statement of Lenz's law is :

The direction of the induced emf or induced current is such that it opposes the change that is producing it.

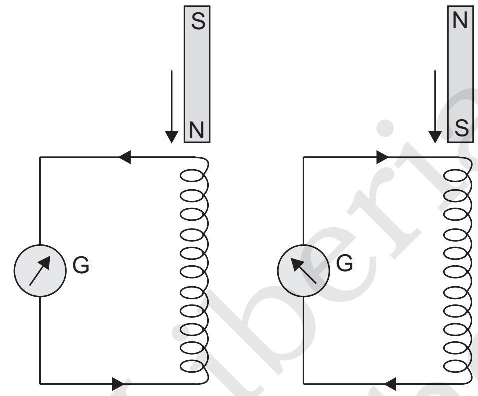
This law may also be stated as under :

The polarity of the induced emf is such that it tends to produce a current which opposes the change that produces it.

When the north pole of the magnet is moved towards the coil (Fig. 3.15 (*a*)), the direction of the induced current in the coil will be such that the upper face of the coil acquires north polarity. So, the coil repels the magnet. In other words, the coil opposes the motion of the magnet towards itself which is really the cause of the induced current in the coil. Similarly, if the south pole of a magnet is moved towards the coil (Fig. 3.15 (*b*)), the upper face of the coil will acquire south polarity thereby opposing the motion of the magnet.

Lenz's Law and Law of Conservation of Energy

According to Lenz's law, the induced emf opposes the change that produces it. It is this opposition against which we perform mechanical work in causing the change in magnetic flux. So, it is the mechanical energy which is converted into electrical energy. Thus, *Lenz's law is in accordance with the law of conservation of energy.*



(a) N pole approaching the coil (b) S pole approaching the coil

Fig. 3.14

If, however, the reverse would happen, then a little change of flux would produce an induced electric current which would help the change of flux further thereby producing more electric current. The increased emf would then cause further change of flux and it would further increase the current and so on. This would create energy out of nothing. It would then violate the law of conservation of energy.

3.12 MOTIONAL ELECTROMOTIVE FORCE

(*Induced emf by changing A*)

Induced emf produced by changing the area of a closed circuit by the movement of the circuit or a part of it through a steady magnetic field is known as motional emf.

Consider a straight conductor moving in a uniform and time-independent magnetic field. Fig. 3.15 shows a rectangular conductor PQRS in which the conductor PQ is free to move. The rod PQ is moved towards the left with a constant velocity v as shown in the figure. Assume that there is no loss of energy due to friction. PQRS forms a closed circuit enclosing an area that changes as PQ moves. It is placed in a uniform magnetic field $\vec{B}$ which is perpendicular to the plane of this system. If the length RQ = x and RS = l, the magnetic flux Φ_B enclosed by the loop PQRS will be

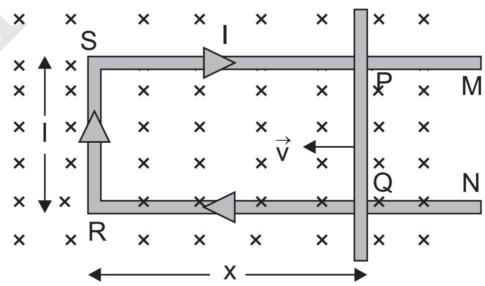


Fig. 3.15. The arm PQ is moved to the left side, thus decreasing the area of the rectangular loop. This movement induces a current I as shown.

$$\Phi_B = Blx$$

Since x is changing with time therefore the time rate of change of magnetic flux Φ_B will induce an emf given by :

$$\varepsilon = \frac{-d\Phi_B}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = Blv$$

Here we have used $\frac{dx}{dt} = -v$ which is the speed of the conductor PQ.

The induced emf Blv is due to the motion of the conductor. So, it is termed as **'motional emf '.** It disappears as soon as the motion of the conductor stops.

The direction of the motional emf is given by Lenz's law. However, it is more convenient to apply **Fleming's right hand rule** stated as follows :

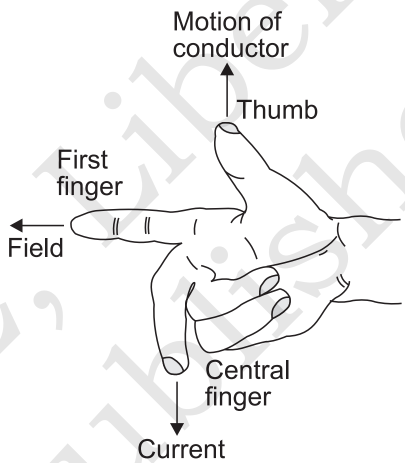


Fig. 3.16. Fleming's right hand rule.

Stretch the thumb and the first two fingers of your right hand in mutually perpendicular directions. If the first finger points in the direction of the magnetic field, the thumb in the direction of motion of the conductor, then the central finger points in the direction of the induced emf or induced current in the conductor [Fig. 3.16].

Applying this rule, we find that the induced current flows from P to Q.

Fleming's right hand rule is also known as **Generator rule.**

Four Important Points

- When a straight conductor of length l moves with constant velocity v in a magnetic field B, the induced emf, $\varepsilon = Blv$ **when B, *l* and *v* are mutually perpendicular**.
- When l or v is parallel to B, then $\varepsilon = 0$.
- Fleming's right hand rule is used to determine the direction of induced current/induced emf.
- Fleming's left hand rule is used to determine the direction of force on a current-carrying conductor.

3.13 EXPLANATION OF MOTIONAL EMF IN TERMS OF LORENTZ MAGNETIC FORCE

It is possible to explain the motional emf in terms of the Lorentz force acting on the free charge carriers of conductor PQ (Fig. 3.16). Consider any arbitrary charge q in the conductor PQ. When the rod moves with speed v, the charge will also be moving with speed v in the magnetic field $\vec{B}$. The Lorentz force on this charge is qvB in magnitude, and its direction is towards Q. All charges experience the same force, in magnitude and direction, irrespective of their position in the rod PQ. The work done in moving the charge from P to Q is,

$$W = qvBl$$

Since emf is the work done per unit charge,

$$\therefore \qquad \varepsilon = \frac{W}{q} = Blv$$

This equation gives emf induced across the rod PQ.

Stationary Conductor in a Changing Magnetic Field

Let us now study how an emf is induced when a conductor is stationary and the magnetic field is changing – a fact which Faraday verified by numerous experiments. In the case of a stationary conductor, the force on its charges is given by

$$\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B}) = q\vec{E} \qquad [\because \ \vec{v} = 0]$$

Thus, any force on the charge must arise from the electric field term $\vec{E}$ alone. Therefore, to explain the existence of induced emf or induced current, we must assume that a time-varying magnetic field generates an electric field. However, it may be noted that the electric fields produced by static electric charges have properties different from those produced by time-varying magnetic fields.

3.14 GALVANOMETER

A galvanometer is a device that is used to detect small electric current or measure its magnitude. The current and its intensity is usually indicated by a magnetic needle's movement or that of a coil in a magnetic field that is an important part of a galvanometer.

Moving Coil Galvanometer

A moving coil galvanometer is an instrument which is used to measure electric currents. It is a sensitive electromagnetic device which can measure how currents even of the order of a few microamperes.

Moving-coil galvanometers are mainly divided into two types:

- Suspended coil galvanometer
- Pivoted-coil or Weston galvanometer

Moving Coil Galvanometer Principle

A current-carrying coil when placed in an external magnetic field experiences magnetic torque. The angle through which the coil is deflected due to the effect of the magnetic torque is proportional to the magnitude of current in the coil.

Construction and Diagram

The moving coil galvanometer is made up of a rectangular coil that has many turns and it is usually made of thinly insulated or fine copper wire that is wounded on a metallic frame. The coil is free to rotate about a fixed axis. A phosphor-bronze strip that is connected to a movable torsion head is used to suspend the coil in a uniform radial magnetic field.

Essential properties of the material used for suspension of the coil are conductivity and a low value of the torsional constant. A cylindrical soft iron core is symmetrically positioned inside the coil to improve the strength of the magnetic field and to make the field radial. The lower part of the coill is attached to a phosphor-bronze spring having a small number of turns. The other end of the spring is connected to binding screws.

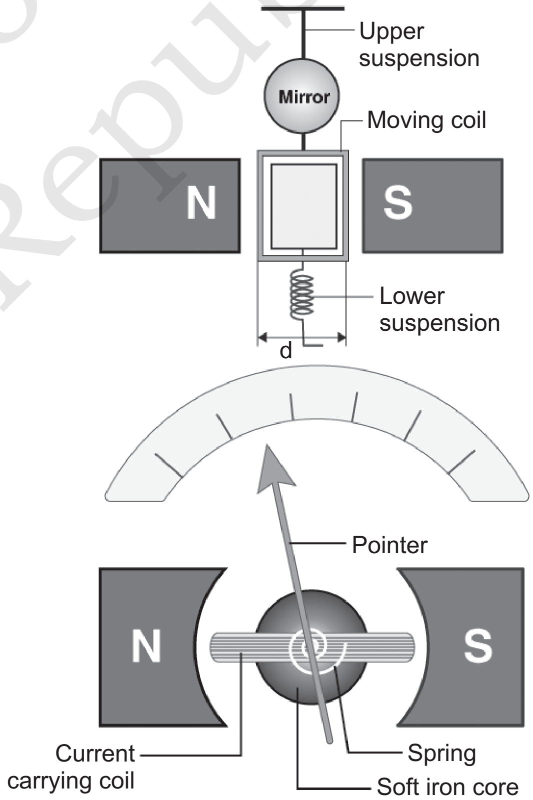


Fig. 3.17. Moving coil galvanometer

The spring is used to produce a counter torque which balances the magnetic torque and hence helps in producing a steady angular deflection. A plane mirror which is attached to the suspension wire, along with a lamp and scale arrangement, is used to measure the deflection of the coil. Zero-point of the scale is at the centre.

Working of Moving Coil Galvanometer

Suppose a current I flows through the rectangular coil of a number of turns and a cross-sectional area A. When this coil is placed in a uniform radial magnetic field B, the coil experiences a torque T.

Let us first consider a simple turn ABCD of the rectangular coil having a length l and breadth b. This is suspended in a magnetic field of strength B such that the plane of the coil is parallel to the magnetic field. Since the sides AB and DC are parallel to the direction of the magnetic field, they do not experience any effective force due to the magnetic field. The sides AD and BC being perpendicular to the direction of field experience an effective force F given by $F = BIl$.

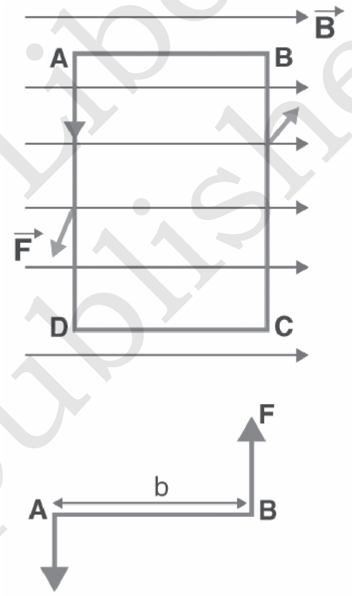


Fig. 3.18

Using Fleming's left-hand rule we can determine that the forces on AD and BC are in opposite directions to each other. When equal and opposite forces F called couple acts on the coil, it produces a torque. This troque causes the coil to deflect.

We know that torque τ = force × perpendicular distance between the forces

$$\tau = F \times b$$

Substituting the value of F we already know,

Torque τ acting on single-loop ABCD of the coil $= BIl \times b$

Where $l \times b$ is the area A of the coil,

Hence, the torque acting on n turns of the coil is given by

$$\tau = nIAB$$

The magnetic torque thus produced causes the coil to rotate, and the phosphor bronze strip twists. In turn, the spring S attached to the coil prodcues a counter torque or restoring torque $k\theta$ which results in a steady angular deflection.

Under equilibrium condition:

$$k\theta = nlAB$$

Here k is called the torsional constant of the spring (restoring couple per unit twist). The deflection or twist θ is measured as the value indicated on a scale by a pointer which is connected to the suspension wire.

$$\theta = (nAB/k)l$$

Therefore, $\theta \propto l$

The quantity nAB/k is a constant for a given galvanometer. Hence it is understood that the deflection that occurs the galvanometer is directly proportional to the current that flows through it.

The deflection θ per unit voltage is known as voltage sensitivity θ/V. Dividing both sides by V in the equation $\theta = (nAB/k)l$:

$$\theta/V = (nAB/Vk)l = (nAB/k)(l/V) = (nAB/k)(1/R)$$

R stands for the effective resistance in the circuit.

It is worth nothing that voltage sensitivity = Current sensitivity/ Resistance of the coil. Therefore, under the condition that R remains constant, voltage sensitivity $\propto$ Current sensitivity.

Applications of Galvanometer

The moving coil galvanometer is a highly sensitive instrument due to which it can be used to defect the presence of current in any given circuit.

The galvanometer can be used to measure:

(a) the value of current in the circuit by connecting it in parallel to low resistance.

(b) the voltage by connecting it in series with high resistance.

3.15 ELECTRIC MOTOR (D.C. MOTOR)

D.C. motor works on the principle which states that ***when a rectangular coil is placed in a magnetic field and current is passed through the coil then a force (torque) acts on the coil and due to this torque, the coil rotates continuously. Direction of rotation of the coil is given by Fleming's Left Hand Rule.***

It is a device which converts electrical energy into mechanical energy of rotation.

Main parts of a D.C. motor are shown in Fig. 3.19.

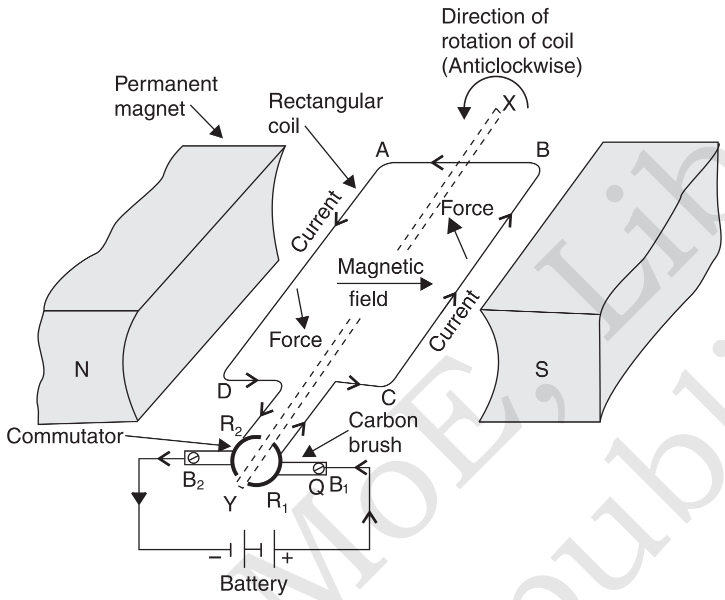


Fig. 3.19. D.C. motor.

DC motor has following **four** main parts:

1. A **permanent magnet** having concave magnetic poles N-S, which provides a strong magnetic field.
2. An **armature,** which is moving part of the motor. It has two parts :
   (*i*) Laminated shaft X-Y and
   (*ii*) Copper coil ABCD wrapped on end X of the shaft inside the field.
3. A **pair of metallic split rings** R_1 and R_2 (commutator).
4. A **pair of metallic or carbon brushes** B_1 and B_2.

Working

A direct current (D.C.) source is connected between metallic brushes B_1 and B_2. When current passes through the coil, it flows in arms DB and AC in a direction perpendicular to the magnetic field. Equal and opposite forces act on these arms (in a direction according to Fleming's Left Hand Rule) and they form a couple. Torque which acts on the coil is given by, τ = NAIB. The coil rotates in anticlockwise direction. After half rotation, split parts of ring change brushes. Current becomes reverse in the arms but couple acts in same direction as before. The coil continues to rotate the shaft on which it is wrapped. Thus, rotatory motion (motor action) becomes available.

3.16 DIRECT CURRENT GENERATOR (DYNAMO)

Electric generator works on the principle that *when a straight conductor is moved in a magnetic field then current, is induced in the conductor.*

It is a device which converts mechanical kinetic energy of rotation into electrical energy.

Main parts of the dynamo and their arrangement is shown in Fig. 3.20.

From Fig. 3.20 we see that it has **four** main parts

1. A **field magnet** having concave magnetic poles N-S, which provides a strong magnetic field.
2. An **armature** which is the moving part of the dynamo. It has two parts.
   (*i*) Laminated shaft XY.
   (*ii*) Copper coil ABCD wrapped on end X of the shaft inside the field.
3. A **pair of metallic split rings** R_1 and R_2 (commutator).
4. A **pair of metallic or carbon brushes** B_1 and B_2.

Working

The shaft is rotated by some mechanical means (strong water current or steam). As the shaft rotates, magnetic flux through the coil changes. This changing magnetic flux produces induced e.m.f. in the coil, whose magnitude changes from zero to maximum and then from maximum to zero, through half the rotation of the coil [Fig. 3.20(a)].

Current flows out from coil into external circuit (load), through B_2, returning through B_1.

During next half rotation, the current would like to change direction. But by then split ring parts exchange brushes. Then B_1 touches R_2 and B_2 touches R_1 [Fig. 3.20(b)].

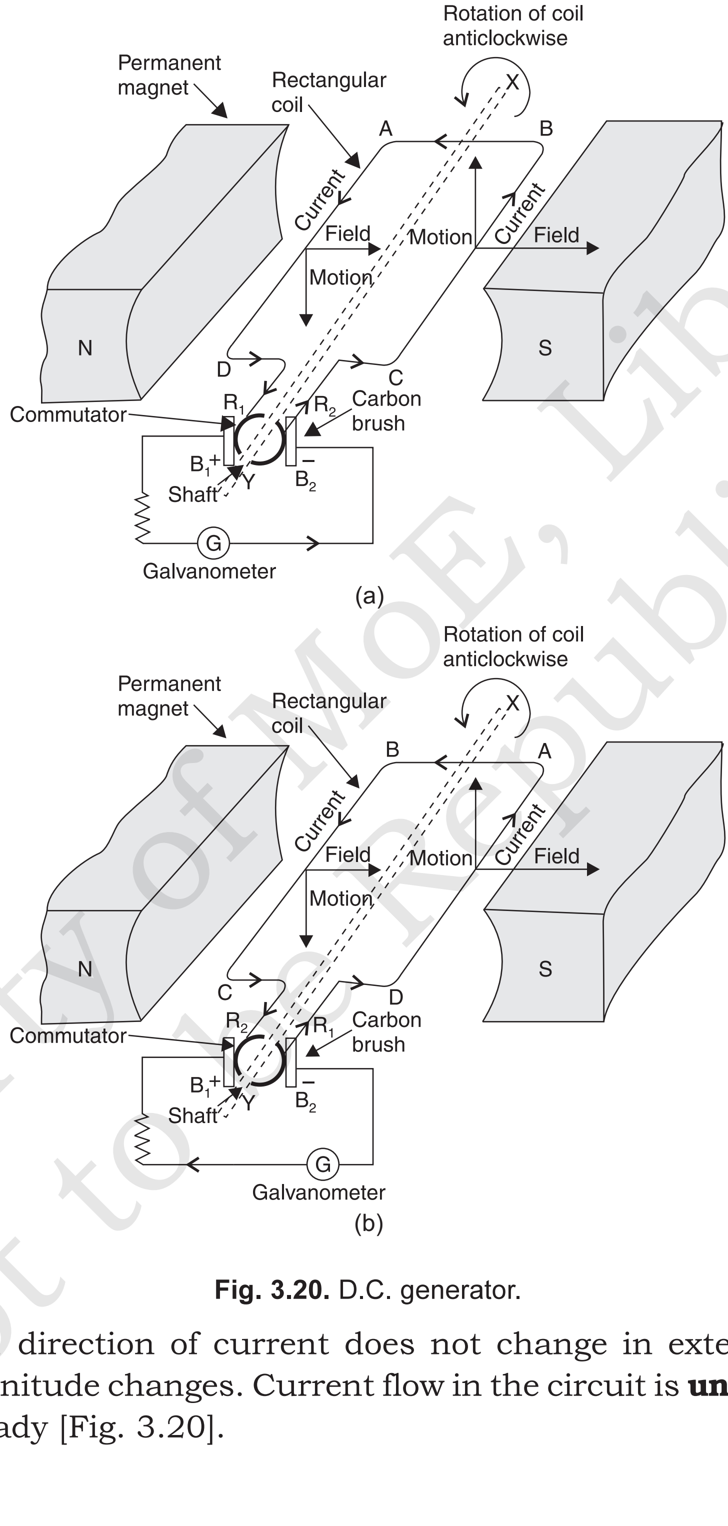


Fig. 3.20. D.C. generator.

Hence, direction of current does not change in external circuit, only its magnitude changes. Current flow in the circuit is **unidirectional** and not steady [Fig. 3.20].

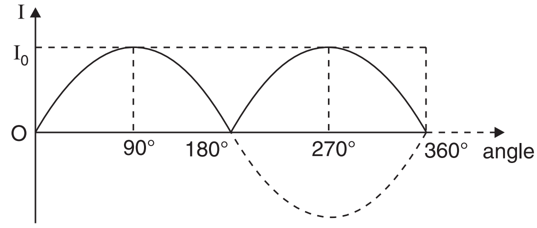


Fig. 3.21. Unidirectional current with single phase D.C. dynamo.

3.17 DIRECT AND ALTERNATING CURRENT

Direct Current (D.C.). A current which has a constant magnitude and same direction, is called a direct current.

Current due to a cell or a battery is a direct current.

Alternating Current (A.C.). A current which changes in magnitude and direction at regular intervals of time is called an alternating current.

Current produced by an A.C. dynamo (A.C. generator) is alternating current. Such a dynamo has separate **slip rings** in place of a **split ring.** Current changes direction after each rotation of the coil.

Frequency. Frequency of A.C. is the number of cycles per second completed by the current. One cycle is completed when the A.C. rises from zero to maximum positive then back to zero and then the maximum negative and zero again.

Remember: These days, most power stations produce AC. In India AC changes directions after every $\frac{1}{100}$ second i.e., frequency of AC produced in India is 50 Hz.

3.18 OPERATION OF AN ALTERNATING CURRENT GENERATOR

The working principle of an alternator or AC generator is similar to the basic working principle of a DC generator.

The following figure will help you understand how an alternator or AC generator works. According to the Faraday's law of electromagnetic induction, whenever a conductor moves in a magnetic field EMF gets induced across the conductor. If the close path is provided to the conductor, induced emf causes current to flow in the circuit.

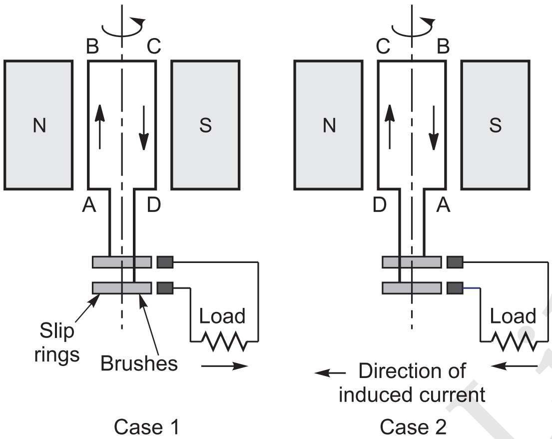


Fig. 3.22. AC Generator

Now, see the above figure. Let the conductor coil ABCD is placed in a magnetic field. The direction of magnetic flux will be from N pole to S pole. The coil is connected to slip rings, and the load is connected through brushes resting on the slip rings.

Now, consider the case 1 from above figure. The coil is rotating clockwise, in this case the direction of induced current can be given by Fleming's right hand rule, and it will be along A-B-C-D.

As the coil is rotating clockwise, after half of the time period, the position of the coil will be as in second case of above figure. In this case, the direction of the induced current according to Fleming's right hand rule will be along D-C-B-A. It shows that, the direction of the current changes after half of the time period, that means we get an alternating current.

3.19 TRANSFORMERS

For many purposes, it is necessary to change (or transform) an alternating voltage from one to another of greater or smaller value. This is done with a device called *transformer* using the principle of mutual induction.

A transformer consists of two sets of coils, insulated from each other. They are wound on a soft-iron core, either one on top of the other as in Fig. 3.23(*a*) or on separate limbs of the core as in Fig. 3.23(*b*). One of the coils called the *primary coil* has N_p turns. The other coil is called the *secondary coil*; it has N_s turns. Often the primary coil is the input coil and the secondary coil is the output coil of the transformer.

When an alternating voltage is applied to the primary, the resulting current produces an alternating magnetic flux which links the secondary and induces an emf in it. The value of this emf depends on the number of turns in the secondary. We consider an ideal transformer in which the primary has negligible resistance and all the flux in the core links both primary and secondary windings. Let ϕ be the flux in each turn in the core at time t due to current in the primary when a voltage v_p is applied to it.

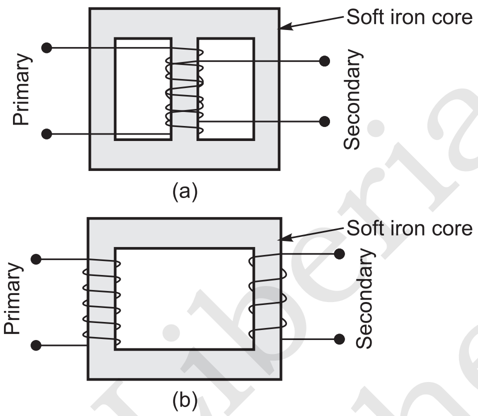


Fig. 3.23. Two arrangements for winding of primary and secondary coil in a transformer: (*a*) two coils on top of each other, (*b*) two coils on separate limbs of the core.

Then the induced emf or voltage ε_s, in the secondary with N_s turns is

$$\varepsilon_s = -\mathrm{N}_s \frac{d\phi}{dt} \qquad [1]$$

The alternating flux ϕ also induces an emf, called back emf in the primary. This is

$$\varepsilon_p = -\mathrm{N}_p \frac{d\phi}{dt} \qquad [2]$$

But $\varepsilon_p = v_p$. If this were not so, the primary current would be infinite since the primary has zero resistance (as assumed). If the secondary is an open circuit or the current taken from it is small, then to a good approximation

$$\varepsilon_s = v_s$$

Where v_s is the voltage across the secondary. Therefore, Eqs. [1] and [2] can be written as

$$v_s = -\mathrm{N}_s \frac{d\phi}{dt} \qquad [1\ (a)]$$

$$v_p = -\mathrm{N}_s \frac{d\phi}{dt} \qquad [2\ (a)]$$

From Eqs. [1 (*a*)] and [2 (*a*)], we have

$$\frac{v_s}{v_p} = \frac{N_s}{N_p} \qquad [3]$$

Note that the above relation has been obtained using three assumptions: (*i*) the primary resistance and current are small; (*ii*) the same flux links both the primary and the secondary as very little flux escapes from the core, and (*iii*) the secondary current is small.

If the transformer is assumed to be 100% efficient (no energy losses), P = IV,

$$I_p V_p = I_s V_s \qquad [4]$$

Although some energy is always lost, this is a good approximation, since a well designed transformer may have an efficiency of more than 95%. Combining Eqs. [3] and [4], we have

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} \qquad [5]$$

Since I and V both oscillate with the same frequency as the ac source, Eq. [5] also gives the ratio of the amplitudes or rms values of corresponding quantities.

Now, we can see how a transformer affects the voltage and current we have.

$$V_s = \left(\frac{N_s}{N_p}\right) V_p \quad \text{and} \quad I_s = \left(\frac{N_p}{N_s}\right) I_p \qquad [6]$$

That is, if the secondary coil has a greater number of turns than the primary ($N_s > N_p$), the voltage is stepped up ($V_s > V_p$). This type of arrangement is called a *step-up transformer*.

If the secondary coil has less turns than the primary ($N_s < N_p$), we have a *step-down transformer*. In this case, $V_s < V_p$ and $I_s > I_p$. That is, the voltage is stepped down, or reduced, and the current is increased.

REVIEW EXERCISE

A. MULTIPLE CHOICE QUESTIONS (MCQs)

1. The fact that magnetic field is produced around a wire carrying a current, was discovered by

(*a*) Faraday (*b*) Oersted
(*c*) Maxwell (*d*) Joule

2. When current is straight, the associated magnetic field is

(*a*) straight (*b*) elliptical
(*c*) circular (*d*) parabolic

3. When current is circular, the associated magnetic field is

(*a*) straight (*b*) elliptical
(*c*) circular (*d*) parabolic

4. When current flows clockwise in a loop, the polarity of its face is

(*a*) east (*b*) south
(*c*) west (*d*) north

5. When current flows anticlockwise in a loop the magnetic polarity of the face is

(*a*) east (*b*) south
(*c*) west (*d*) north

6. For a solenoid carrying a current I and having n turns per unit length, wrapped on a core of permeability μ, the correct expression for magnetic field intensity (B) is

(*a*) $B = \frac{\mu_0}{\mu} nI$

(*b*) $B = \frac{\mu_0 \mu I}{n}$

(*c*) $B = \mu_0 \mu n I$

(*d*) $B = \frac{\mu_0 \mu n}{I}$

7. Magnets having temporary magnetism are called

(*a*) electromagnets
(*b*) bar magnets
(*c*) circular magnets
(*d*) horse-shoe magnets.

8. Direction of force acting on a current carrying conductor kept in a magnetic field is given by

(*a*) Fleming's right hand rule

(*b*) Fleming's left hand rule

(*c*) Lenz's rule

(*d*) Faraday's rule.

9. The electric device which works on the phenomenon of force on a current carrying conductor in a magnetic field is

(*a*) generator

(*b*) accelerator

(*c*) motor

(*d*) transformer.

10. The moving part of an electric motor is called

(*a*) armature

(*b*) shaft

(*c*) slip ring

(*d*) split ring.

11. Electromagnetic induction was discovered by

(*a*) Oersted

(*b*) Maxwell.

(*c*) Thomson

(*d*) Faraday.

12. Direction of induced current produced by motion of a conductor in a magnetic field is given by

(*a*) Fleming's right hand rule

(*b*) Fleming's left hand rule

(*c*) Lenz's rule

(*d*) Faraday's rule.

13. In domestic electric circuits, fuse must be placed in series with

(*a*) earth wire

(*b*) neutral wire

(*c*) live wire

(*d*) any of the three wires.

14. Meter and main switch is contained in a main board fitted usually
(*a*) at street electric pole
(*b*) at main gate of building
(*c*) in varandah or poarch
(*d*) in bed or study room.

15. High powered electrical appliances are earthed to
(*a*) avoid shock
(*b*) avoid wastage
(*c*) make the appliance look beautiful
(*d*) reduce the bill.

B. FILL IN THE BLANKS

1. The phenomenon of production of magnetic field round a current carrying conductor is called ______ effect of current.
2. The rule which relates direction of deflection of magnetic needle with direction of field is called ______ rule.
3. When a wire is wrapped into many close turns over a cylindrical core, it forms a ______ .
4. To have north polarity at a face, the current in loop must flow in ______ direction.
5. To have south polarity at a face, the current in loop must flow in ______ direction.
6. In electromagnets, magnetism is ______ .
7. In an electric motor ______ energy is converted into mechanical energy.
8. In electromagnetic induction, motion of a ______ in a fixed coil produces electric current.
9. In an electric generator, ______ energy is converted into an electrical energy.
10. An electric current having a constant magnitude and direction, is called a ______ current.

C. VERY SHORT ANSWER TYPE QUESTIONS

1. What does the divergence of magnetic field lines near the ends of a current-carrying straight solenoid indicate?

2. Name three appliances wherein an electric motor, a rotating device that converts electrical energy to mechanical energy, is used as an important component. In what respect motors are different from generators?

3. A magnetic compass shows a deflection when placed near a current-carrying wire. How will the deflection of the compass get affected if the current in the wire is increased? Support your answer with a reason.

4. It is established that an electric current through a metallic conductor produces a magnetic field around it. Is there a similar magnetic field produced around a thin beam of moving (*i*) alpha particles, (*ii*) neutrons? Justify your answer.

5. What is the ratio of SI to CGS units of magnetic induction?

6. Why is soft-iron not used for making a permanent magnet?

7. Name the rule to find the direction of force on a current-carrying conductor placed in direction perpendicular to the direction of magnetic field.

8. What is the relation between a tesla, an ampere and a meter.

9. What is the force acting on charge (q) moving in a direction perpendicular to a magnetic field (B) with velocity v?

10. What does the direction of thumb indicate in the right-hand thumb rule? In what way this rule is different from Fleming's left-hand rule?

D. SHORT ANSWER TYPE QUESTIONS

1. A student performs an experiment to study the magnetic effect of current around a current carrying straight conductor. He reports that

(i) the direction of deflection of the north pole of a compass needle kept at a given point near the conductor remains unaffected even when the terminals of the battery sending current in the wire are interchanged.

(*ii*) for a given battery, the degree of deflection of a N-pole decreases when the compass is kept at a point farther away from the conductor. Which of the above observations of the student is incorrect and why?

2. A current carrying conductor is placed perpendicular to the magnetic field of horse-shoe magnet. The conductor is displaced upward. What will happen to the displacement of the conductor if (*i*) current in the conductor is increased, (*ii*) a horse-shoe magnet is replaced by another stronger horse-shoe magnet and (*iii*) the length of the conductor is increased.

3. What is frequency of a direct current?

4. Can a 12 volt battery be used to operate a step-up transformer?

5. Can alternating current be used to perform electrolysis?

E. LONG ANSWER TYPE QUESTIONS

1. What does the direction of thumb indicate in the right-hand thumb rule? In what way this rule is different from Fleming's left-hand rule?

2. Meena draws magnetic field lines of field close to the axis of a current-carrying circular loop. As she moves away from the centre of the circular loop, she observes that the lines keep on diverging. How will you explain her observation?

3. What does the divergence of magnetic field lines near the ends of a current-carrying straight solenoid indicate?

4. Name four appliances wherein an electric motor, a rotating device that converts electrical energy to mechanical energy, is used as an important component. In what respect motors are different from generators?

5. What is the role of the two conducting stationary brushes in a simple electric motor?

SEMESTER-TWO

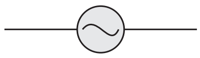
PERIOD-4

P12CH4

TOPIC 4 Alternating Current (AC) and Electronics

4.1 ALTERNATING CURRENT

Alternating current *is that current which continuously varies in magnitude and periodically reverses its direction.* The same is true of alternating voltage.

> *Symbol of ac source*
>
> 
>
> *Note that polarity is not marked on ac source.*

During one cycle, the current (or voltage) first rises from zero to maximum in one direction, falls to zero, then becomes maximum in the reverse direction and again falls to zero.

4.2 AVERAGE VALUE OF AC OVER HALF CYCLE

Average or mean value of alternating current over half cycle *is that steady current which will send the same amount of charge in a circuit in the time of half cycle* $\left(i.e., \frac{T}{2}\right)$ *as is sent by the given alternating current in the same circuit in the same time* $\left(i.e., \frac{T}{2}\right)$.

Let the alternating current be represented by $i = i_m \sin \omega t$.

Mean value of ac, $I_{mean} = \frac{2}{\pi} i_m = \mathbf{0.637\ i_m}$ or $\boxed{I_{mean} = 63.7\%\ i_m}$

So, the mean value of alternating current for half cycle is 0.637 times the peak value of ac. [The same is true of the alternating voltage.]

4.3 ROOT MEAN SQUARE VALUE AND PEAK VALUE OF ALTERNATING CURRENT

Root Mean Square value (rms value) or Virtual value or Effective value of ac *is that steady current which would produce the same heat in given resistance in a given time as is done by the alternating current when passed through the same resistance for the same time. It is represented by I or I_{rms} or I_v or I_{eff}.*

Let the alternating current be represented by $i = i_m \sin \omega t$.

We have, $$I = \frac{i_m}{\sqrt{2}} = 0.707\, i_m$$

The root mean square or virtual value of alternating current is 0.707 times the peak value of alternating current.

Example 1: *The peak value of an alternating current is 5 A and its frequency is 60 Hz. Find its rms value.*

Solution: $$I_{rms} = \frac{i_m}{\sqrt{2}} = \frac{5}{\sqrt{2}}\text{ A} = \mathbf{3.536\ A}$$

Example 2: *In the previous question, how long will the current take to reach the peak value, starting from zero?*

Solution: $$T = \frac{1}{\nu} = \frac{1}{60}\text{ s}$$

$$t = \frac{T}{4} = \frac{1}{4 \times 60}\text{ s} = \frac{1}{240}\text{ s} = \mathbf{4.167\ ms}$$

Example 3: *The instantaneous current from an ac source is I = 6 sin 314t. What is the rms value of the current ?*

Solution: Comparing the given equation with $i = i_m \sin \omega t$, we get

$$i_m = 6\text{ A}$$

$$I_{rms} = \frac{i_m}{\sqrt{2}} = \frac{6}{\sqrt{2}}\text{ A} = 3\sqrt{2}\text{ A} = \mathbf{4.24\ A}$$

4.4 AC VOLTAGE APPLIED TO A RESISTOR

Figure 4.1 shows a resistor connected to a source ε of ac voltage. The symbol for an ac source in a circuit diagram is (~). We consider a source which produces sinusoidally varying potential difference across its terminals. Let this potential difference, also called ac voltage, be given by

$$v = v_m \sin \omega t \qquad ...(1)$$

where v_m is the amplitude of the oscillating potential difference and ω is its angular frequency.

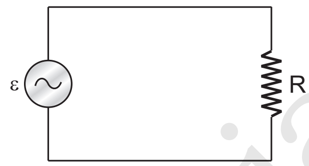


Fig. 4.1. AC voltage applied to a resistor.

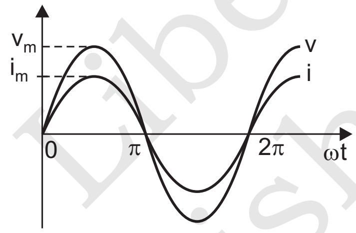


Fig. 4.2. In a pure resistor, the voltage and current are in phase. The minima, zero and maxima occur at the same respective times.

Applying Kirchhoff's loop rule to the circuit shown in Fig. 4.1, we get

$$v_m \sin \omega t = i\mathrm{R}$$

or $$i = \frac{v_m}{\mathrm{R}} \sin \omega t$$

or $$i = i_m \sin \omega t \qquad ...(2)$$

where the current amplitude i_m is given by

$$i_m = \frac{v_m}{\mathrm{R}} \qquad ...(3)$$

Equation (3) is just Ohm's law which for resistors works equally well for both ac and dc voltages. The voltage across a pure resistor and the current through it, given by Eqs. (1) and (2) are plotted as a function of time in Fig. 4.2. Note, in particular that both v and i reach zero, minimum and maximum values at the same time. Clearly, *the voltage* and *current are in phase with each other.*

4.5 REPRESENTATION OF AC CURRENT AND VOLTAGE BY ROTATING VECTORS–PHASORS

We know that the current through a resistor is in phase with the voltage. But this is not so in the case of an inductor, a capacitor or a combination of these circuit elements. In order to show phase relationship between voltage and current in an ac circuit, we use the notion of *phasors.* A phasor is a vector which rotates about the origin with angular speed ω as shown in Fig. 4.3. The vertical components of phasors $\vec{\mathrm{V}}$ and $\vec{\mathrm{I}}$ represent the sinusoidally varying quantities v and i. Figure 4.3 shows the voltage and current phasors and their relationship at time t_1 for the case of an ac source connected to a resistor *i.e.*, corresponding to the purely resistive circuit. From Fig. 4.3 we see that phasors $\vec{V}$ and $\vec{I}$ for the case of a resistor are in the same direction. This is so for all times. This means that the phase angle between the voltage and the current is zero.

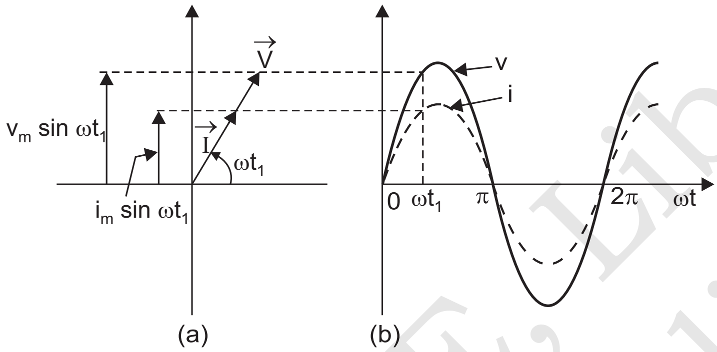


Fig. 4.3. (*a*) A phasor diagram for the purely resistive circuit (*b*) Graph of *v* and *i* versus ω*t*.

4.6 INDUCTIVE REACTANCE (X_L)

We know that $i_m = \dfrac{v_m}{\omega L}$

Comparing with Ohm's law equation $\left(I = \dfrac{V}{R}\right)$, we find that ωL plays the same role in ac circuit as is played by R in dc circuit. The term ωL is called *inductive reactance.* The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit.

Inductive reactance is defined as the opposition offered by an inductor to the flow of alternating current through it. It is denoted by X_L.

$$X_L = \omega L = 2\pi f L$$

where f is the frequency of ac supply. *The inductive reactance is directly proportional to the inductance and to the frequency of the current.*

For a given coil, L is constant. $\therefore \quad X_L \propto f$

So, higher the frequency of ac supply, greater will be the value of inductive reactance Fig. 4.4.

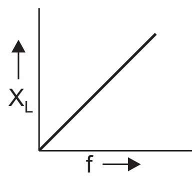


Fig. 4.4. Variation of X_L with f.

The dimensions of inductive reactance are the same as those of resistance. The SI unit of inductive reactance is ohm (Ω).

Example 4: *A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.*

Solution:

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3}\ \Omega$$

$$= 7.85\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220\ \text{V}}{7.85\ \Omega} = \mathbf{28.03\ A}$$

Example 5: *Calculate the value of the current through an inductance of 1 henry and of negligible resistance when connected to an ac source of 200 V and 50 Hz.*

Solution:

$$I_\upsilon = \frac{V_\upsilon}{\omega L} = \frac{V_\upsilon}{2\pi f L} = \frac{200}{2 \times 3.14 \times 50 \times 1}\text{A}$$

$$= \frac{220}{314}\text{A} = \mathbf{0.637\ A}$$

4.7 AC VOLTAGE APPLIED TO A CAPACITOR

Figure 4.5 shows an ac source ε connected to a capacitor only, a purely capacitive ac circuit.

It generates a voltage given by:

$$v = v_m \sin \omega t \qquad ...(1)$$

When a capacitor is connected to a voltage source in a dc circuit, current will flow for the short time required to charge the capacitor. As charge accumulates on the capacitor plates, the voltage across them increases, opposing the current. That is, a capacitor in a dc circuit will limit or oppose the current as it charges. When the capacitor is fully charged, the current in the circuit falls to zero.

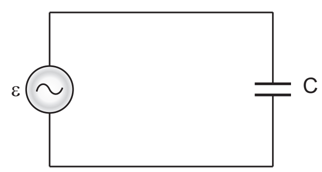


Fig. 4.5. An ac source connected to a capacitor.

When the capacitor is connected to an ac source, as in Fig. 4.5, it limits or regulates the current, but does not completely prevent the flow of charge. The capacitor is alternately charged and discharged as the current reverses each half cycle.

We can write,

$$i_m = \frac{v_m}{(1/\omega C)}$$

The quantity $\frac{1}{\omega C}$ is analogous to the resistance. It is called *capacitive reactance* and is denoted by X_C.

$$X_C = 1/\omega C$$

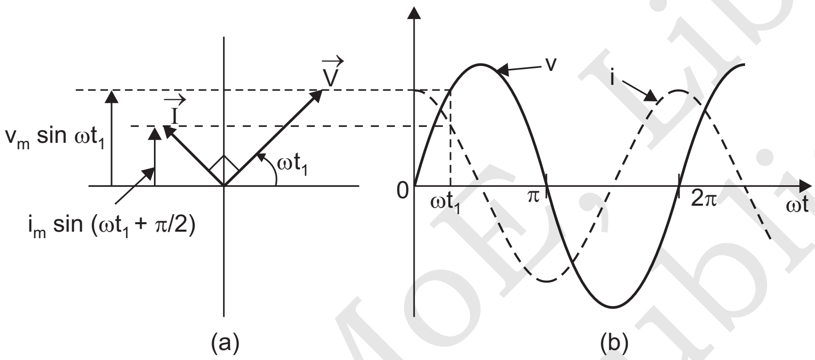


Fig. 4.6. (*a*) A phasor diagram for the purely capacitive circuit (*b*) Graph of v and i versus ωt.

The amplitude of the current is

$$i_m = \frac{v_m}{X_C}$$

Figure 4.6(a) shows the phasor diagram at an instant t_1. Here the current phasor $\vec{I}$ is $\pi/2$ ahead of the voltage phasor $\vec{V}$ as they rotate counterclockwise. Figure 4.6(*b*) shows the variation of voltage and current with time. We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period.

4.8 CAPACITIVE REACTANCE

We know that $$i_m = \frac{v_m}{\frac{1}{\omega C}}$$

Comparing this equation with Ohm's law equation, we find that $\frac{1}{\omega C}$ plays the same role in ac circuit as is played by resistance R in dc circuit. The term $\frac{1}{\omega C}$ is called *capacitive reactance.*

The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit.

Capacitive reactance is defined as the opposition offered by a capacitor to the flow of alternating current through it. It is denoted by X_C.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Capacitive reactance is inversely proportional to the frequency and the capacitance.

For constant C, $X_C \propto \frac{1}{f}$.

So, the capacitive reactance is inversely proportional to frequency [Fig. 4.7].

For direct current, $f = 0$. $\therefore$ $X_C = \infty$

So, a capacitor offers infinite opposition to the flow of direct current through it. In other words, a capacitor blocks dc.

The dimensions of capacitive reactance are the same as those of resistance. The SI unit of capacitive reactance is ohm.

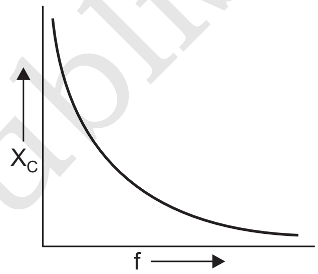


Fig. 4.7. Variation of X_C with *f*.

4.9 AC VOLTAGE APPLIED TO A SERIES LCR CIRCUIT

Figure 4.8 shows a series LCR circuit connected to an ac source ε. This circuit is a series combination of a pure inductance L, an ideal capacitor of capacitance C and a pure resistance R. Let the voltage across the source be :

$$v = v_m \sin \omega t$$

where v_m is the amplitude of the oscillating potential difference and ω is its angular frequency.

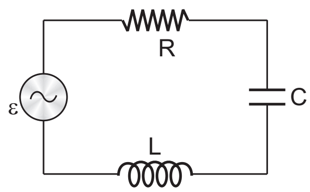


Fig. 4.8. A series LCR circuit connected to an ac source.

If q is the charge on the capacitor and i the current at time t, we have, from Kirchhoff's loop rule:

$$L\frac{di}{dt} + iR + \frac{q}{C} = v \qquad \ldots(1)$$

By analogy to the resistance in a circuit, we introduce the *impedance Z* in an ac circuit:

$$i_m = \frac{v_m}{Z}$$

where $$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \qquad \ldots(5)$$

Impedance of LCR circuit is defined as the effective opposition (resistance) offered by LCR circuit to the flow of alternating current through it.

The reciprocal of impedance is called admittance. It is measured in mho *i.e.,* ohm^{-1} or siemen. Admittance = $\frac{1}{Z}$.

Special Cases

Case I. If $X_C > X_L$, then ϕ is positive and the circuit is predominantly capacitive. Consequently, the current in the circuit leads the source voltage.

Case II. If $X_C < X_L$, then ϕ is negative and the circuit is predominantly inductive. Consequently, the current in the circuit lags the source voltage.

Case III. If $X_L = X_C$, then

$$\tan\phi = 0 \text{ or } \phi = 0°.$$

In this case, the current and the voltage are in the same phase.

Again, $$Z = \sqrt{R^2 + (X_L - X_L)^2} = R$$

So, the LCR circuit behaves like a purely resistive circuit. The impedance is now independent of the frequency of alternating current.

Due to minimum value of impedance, the current in LCR series circuit is maximum. This condition is called **resonance**.

4.10 ADVANTAGES OF ALTERNATING CURRENT OVER DIRECT CURRENT

1. The ac is available in a wide range of voltages. These voltages can be easily stepped up or stepped down with the help of transformers.

2. The cost of generation of ac is less than that of dc.

3. The ac can be conveniently converted into dc with the help of rectifiers.

4. The ac appliances are simple, robust and require less care as compared to dc devices.

5. By supplying ac at high voltages, we can minimise line losses.

4.11 DISADVANTAGES OF ALTERNATING CURRENT OVER DIRECT CURRENT

1. The ac is more dangerous than dc.

2. The maximum voltage of ac is higher than the effective value indicated by ac voltmeter.

3. The ac is transmitted more by the surface of the conductor. This is called *skin effect*. It is for this reason that several strands of thin insulated wire, instead of a simple thick wire, need be used.

4. For electroplating, electrorefining, electrotyping etc., only dc can be used and not ac.

4.12 ENERGY BANDS IN SOLIDS (BAND THEORY OF SOLIDS)

According to Bohr's atomic model, the electrons have well-defined energy levels in an isolated atom. However, if an atom belongs to a crystal, then the energy levels are modified because of the presence of neighbouring atoms. This modification is not appreciable in the case of energy levels of electrons in the inner shells. But it is considerable in the case of the energy levels of the electrons in the outermost shell. This is because the electrons in the outermost shell are shared by more than one atom in the crystal. In order to understand the modification in the energy levels, consider the case of silicon because of the predominance of silicon devices. Its electronic configuration is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^2$.

Consider a single crystal of silicon having N atoms. Each atom in the crystal is situated at a lattice site. In 4.9, the interatomic spacing r is plotted along x-axis and the energy is plotted along y-axis. The distance $r = a \approx 1$ Å corresponds to the actual crystal lattice spacing. It is the equilibrium distance between atoms.

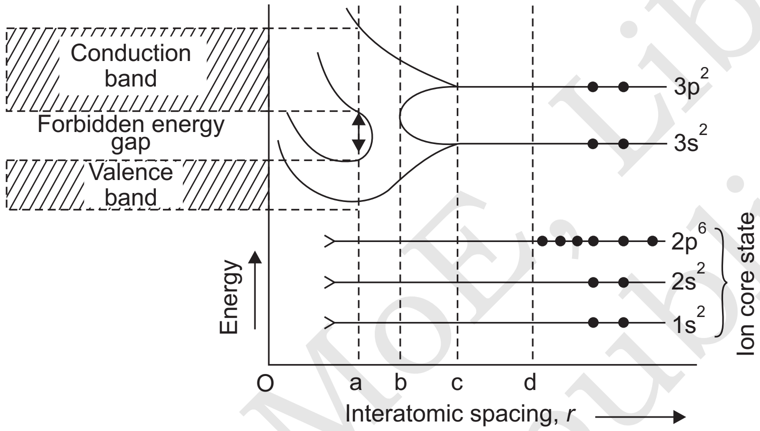


Fig. 4.9. Formation of energy bands in silicon.

In order to understand the process of splitting of energy levels for silicon, let us consider different situations.

(*i*) **When r = d >> a.** In this situation, the interatomic spacing is supposed to be infinite and each atom may be regarded as an isolated atom. Each of N atoms has its own energy levels. The energy levels of the N atoms are identical. The energy levels of each atom remain unmodified. Moreover, the energy levels are sharp, discrete and distinct.

The outer two sub-shells (3*s* and 3*p* sub-shells of the M or $n = 3$ shell) of silicon atom contain two *s* electrons and two *p* electrons. So, in the silicon crystal under consideration, there are 2N electrons completely filling 2N possible 3*s* levels, all of which are at the same energy.

(*ii*) **When c < r < d.** When the interatomic spacing *r* is between *d* and *c*, there is no visible splitting of energy levels. However, there develops a tendency for the splitting of energy levels.

(*iii*) **When r = c.** When the interatomic spacing is equal to *c*, the interaction between the outermost shell electrons of neighbouring silicon atoms becomes appreciable. Consequently, the splitting of the energy levels commences.

(*iv*) **When b < r < c.** The interatomic spacing *r* is between *c* and *b*. The energy of electrons corresponding to the 3*s* and 3*p* levels of each atom gets slightly changed. Instead of a single 3*s* or 3*p* level, we get a large number of closely spaced levels. Corresponding to a single 3*s* level of an isolated atom, we get 2N levels. Similarly, we get 6N levels for a single 3*p* level of an isolated atom. The spreading of energy corresponding to the 3*s* and 3*p* levels reduces the energy gap that existed between the 3*s* and 3*p* levels of an isolated atom. This energy gap is called *Forbidden energy gap.*

Since N is a very large number ($\approx 10^{29}$ atoms/m^3) and energy of 3*s* and 3*p* levels is of the order of a few *e*V, therefore, the levels due to the spreading of energy of 3*s* or 3*p* levels are very closely spaced. This collection of very closely spaced levels (spacing $\approx 10^{-23}$ *e*V for 1 cm^3 crystal) is called an energy band.

(*v*) **When r = b.** When the interatomic spacing is equal to *b*, the energy gap between 3*s* and 3*p* levels disappears completely. The 8N energy levels are continuously distributed. It is not possible to distinguish between the electrons belonging to 3*s* and 3*p* subshells.

(*vi*) **When r = a.** When the interatomic spacing is equal to a (actual spacing in the crystal), the band of 8N levels splits into two bands, band of 4N energy levels completely filled with electrons and the band of 4N unfilled levels. The lower energy band of 4N filled levels is called the **valence band**. The upper band of 4N empty levels is called the **conduction band**. The gap between top of valence band and bottom of conduction band is called **forbidden gap** or **energy gap** or **band gap**. No allowed energy levels for electrons can exist in the forbidden gap.

4.13 ENERGY BANDS IN CONDUCTORS, SEMICONDUCTORS AND INSULATORS (*QUALITATIVE IDEAS ONLY*)

Depending upon the energy gap between valence band and conduction band, the solids behave as conductors, semiconductors or insulators as explained below:

1. Conductors. The energy band structure in conductors has two possibilities :

(*i*) There is an extremely small energy gap between the completely filled valence band and the partially filled conduction band

[Fig. 4.10 (*a*)]. This band structure, is met in alkali metals (Li, Na, K etc.), noble metals (Cu, Ag, Au) and third group elements like Al, Ga, In and Tl.

(*ii*) The valence band is completely filled and the conduction band is empty but the two partially overlap each other [Fig. 4.10(*b*)]. This band structure is seen in metals like Be, Mg, Zn etc.

In both the situations, it can be assumed that there is a single energy band which is partially filled. Therefore, on applying even a small electric field, the conductors conduct electricity.

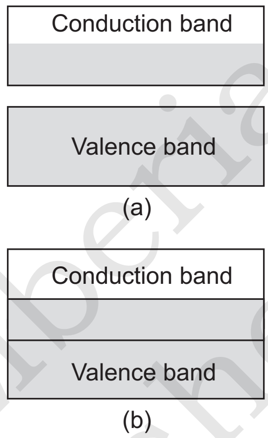


Fig. 4.10

The highest energy level occupied by electrons in a crystal, at absolute zero of temperature, is called Fermi level. The energy corresponding to this level is called the Fermi energy. Under an applied field, the electrons get enough energy to go beyond Fermi energy and thus permit electrical conduction to take place.

2. Semiconductors. Semiconductors are solids for which valence band is completely filled and conduction band is completely empty. There is a small band gap (Eg < 3 eV) between the valence band and the conduction band [Fig. 4.11]. For germanium, band gap is 0.72 eV and for silicon it is 1.1 eV.

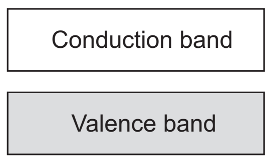


Fig. 4.11

At absolute zero of temperature, electrons are not able to cross the band gap. So, conduction band is totally empty. Thus, no current can flow in semiconductors at 0 K and they behave like insulators.

At room temperature, some valence electrons acquire thermal energy greater than the energy gap. They move to the conduction band where they are free to move under the influence of even a weak electric field. This is the specific property of the crystal which is known as a semiconductor. Higher the temperature, greater are the chances of electrons to jump to conduction band and greater is the conductivity. Clearly, the resistance of semiconductors decreases with increase in temperature.

3. Insulators. In insulators, the valence band is completely filled while the conduction band is empty. There is a large energy gap (Eg > 3 eV) between the valence and conduction bands [Fig. 4.12]. For example, in case of diamond, the energy gap is of 6 eV. Since energy gap is large therefore no electron is able to go from valence band to the conduction band even if electric field is applied. Thus, electrical conduction is not possible through an insulator.

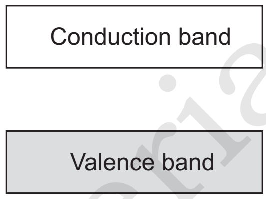


Fig. 4.12

4.14 ELECTRONS AND HOLES IN SEMICONDUCTORS

Figure 4.13 shows the energy band diagram of an intrinsic semiconductor (pure semiconductor). Fig. 4.13 (*a*) shows some charge carriers at absolute zero of temperature. Fig. 4.13 (*b*) shows the situation at room temperature. At room temperature, some electrons are in conduction band leaving an equal number of holes (o) in valence band. Note that each horizontal line represents an energy level. In each energy level, there can be at the most two electrons.

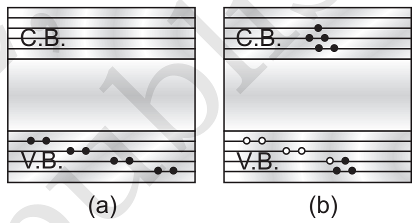


Fig. 4.13. Electrons and holes in semiconductors.

Those electrons in an intrinsic semiconductor which move to the conduction band at high temperatures are called intrinsic carriers. In the valence band, a vacancy is created at the place where the electron was present before moving to the conduction band. This vacancy is called a hole.

The creation of a hole can also be understood by referring to Fig. 4.14. On receiving an additional energy, one of the electrons contributing to a covalent bond breaks and is free to move in the crystal lattice. While coming out of the covalent bond, it leaves behind a hole which is shown as an open circle. An electron from the neighbouring atom can break away

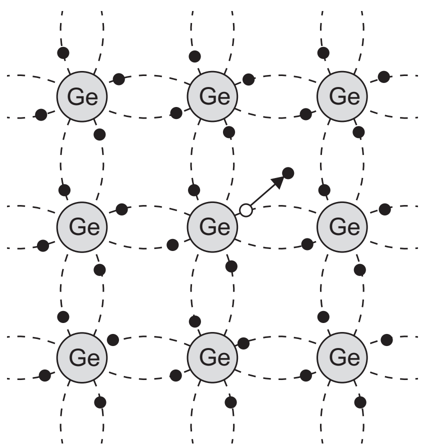


Fig. 4.14. Creation of a hole.

and can come to the place of the missing electron (or hole) completing the covalent bond and creating a hole at another place. The holes move randomly in a crystal lattice.

The completion of a bond may not be necessarily due to an electron from a bond of a neighbouring atom. The bond may be completed by a conduction band electron i.e., free electron and this case is referred to as electron-hole recombination.

The breaking of bonds or generation of electron-hole pairs, and completion of bonds due to recombination is taking place all the time. At equilibrium, the rate of generation becomes equal to the rate of recombination, giving a fixed number of free electrons and holes.

4.15 INTRINSIC SEMICONDUCTOR

It is a pure semiconductor *i.e.,* it is free from impurities. The energy gap in the case of silicon is 1.1 eV. In the case of Germanium, it is 0.74 eV.

Silicon and Germanium are the best examples of semiconductors because these are the most widely used semiconductors.

In intrinsic semiconductors, the number of free electrons, n_e is equal to the number of holes, n_h. That is

$$n_e = n_h = n_i \qquad ...(1)$$

where n_i is called intrinsic carrier concentration.

Semiconductors posses the unique property in which, apart from electrons, the holes also move.

An intrinsic semiconductor will behave like an insulator at T = 0 K as shown in Fig. 4.15(*a*). It is the thermal energy at higher temperatures (T > 0 K), which excites some electrons from the valence band to the conduction band. These thermally excited electrons at T > 0 K, partially occupy the conduction band. Therefore, the energy-band diagram of an

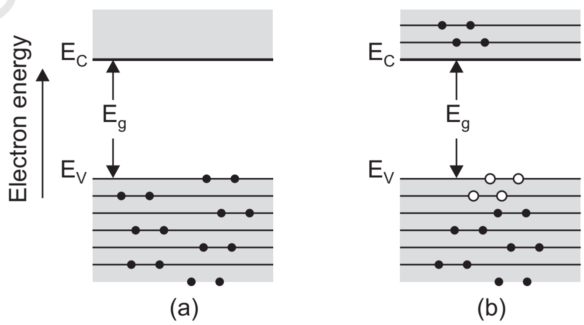


Fig. 4.15. (*a*) An intrinsic semiconductor at T = 0 K behaves like insulator. (*b*) At T > 0 K, four thermally generated electron-hole pairs. The filled circles (•) represent electrons and empty circles (○) represent holes.

intrinsic semiconductor will be as shown in Fig. 4.15(*b*). Here, some electrons are shown in the conduction band. These have come from the valence band leaving equal number of holes there.

4.16 EXTRINSIC SEMICONDUCTORS

*An **extrinsic semiconductor** is one in which an impurity with a valency higher or lower than the valency of the semiconductor atoms is deliberately introduced, thereby drastically influencing the electrical properties of the semiconductor.*

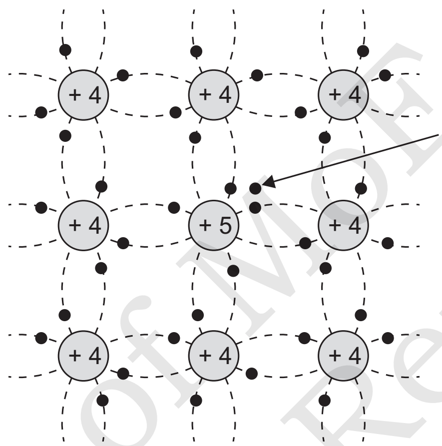


Fig. 4.16. Pentavalent donor atom (As, Sb, P, etc.) doped for tetravalent Si or Ge giving *n*-type semiconductor.

When a small amount, say, a few parts per million (ppm), of a suitable impurity is added to the pure semiconductor, the conductivity of the semiconductor is increased manifold. Such materials are known as *extrinsic semiconductors* or *impurity semiconductors*. The deliberate addition of a desirable impurity is called *doping* and the impurity atoms are called *dopants*. Such a material is also called a *doped semiconductor*.

There are two types of dopants used in doping the tetravalent Si or Ge:

(*i*) Pentavalent (valency 5); like Arsenic (As), Antimony (Sb), Phosphorous (P), etc.

(*ii*) Trivalent (valency 3); like Indium (In), Boron (B), Aluminium (Al), etc.

Depending upon the type of dopant present in the semiconductor, an extrinsic semiconductor may be classified as *n*-type or *p*-type.

4.16.1 n-Type Semiconductor

When an elemental semiconductor of Group IV such as Si or Ge is doped with a pentavalent impurity (an element of group V such as phosphorus, arsenic or antimony), we get n-type semiconductor.

Si or Ge with a pentavalent element is shown in Fig. 4.16. When an atom of +5 valency element occupies the position of an atom in the crystal lattice of Si, four of its electrons bond with the four silicon neighbours while the fifth remains very weakly bound to its parent atom.

In the energy band picture, the energy state corresponding to the fifth valence electron is in the forbidden gap and is slightly below the conduction band. The energy level is indicated by the dashed line in Fig. 4.17. This level is called the *donor level.*

When the *fifth valence electron is transferred to the conduction band,* the parent impurity atom *becomes positively charged* immobile ion. In this way, each impurity atom donates a free electron to the semiconductor. The pentavalent dopant is donating one extra electron for conduction and hence is known as *donor* impurity. It is for this reason that *n*-type semiconductor is sometimes called donor-type semiconductor.

The number of electrons made available for conduction by dopant atoms depends strongly upon the doping level and is independent of any increase in ambient temperature. On the other hand, the number of free electrons (with an equal number of holes) generated by Si atoms, increases weakly with temperature.

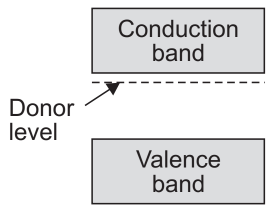


Fig. 4.17. Donor energy level.

Thus, with proper level of doping, the number of conduction electrons can be made much larger than the number of holes. Hence in an extrinsic semiconductor doped with pentavalent impurity, electrons become the *majority carriers* and holes the *minority carriers*. These semiconductors are, therefore, known as *n-type semiconductors*. For *n*-type semiconductors, we have

$$n_e >> n_h$$

4.16.2 p-Type Semiconductor

When an elemental semiconductor of Group IV such as Si or Ge is doped with a trivalent impurity (an element of Group III such as Indium, Boron or Gallium), we get p-type semiconductor.

When Si or Ge is doped with a trivalent impurity like Al, B, In etc, we get *p*-type semiconductor. The dopant has one valence electron less than Si or Ge and, therefore, this atom can form covalent bonds with neighbouring three Si atoms but does not have any electron to offer to the fourth Si atom. So the bond between the fourth neighbour and the trivalent atom has a vacancy or hole as shown in Fig. 4.18. It is obvious that one *acceptor* atom gives one *hole.* These holes are in addition to the intrinsically generated holes while the source of conduction electrons is only intrinsic generation. Thus, for such a material, the holes are the majority carriers and electrons are minority carriers. Therefore, extrinsic semiconductors doped with trivalent impurity are called *p-type semiconductors*.

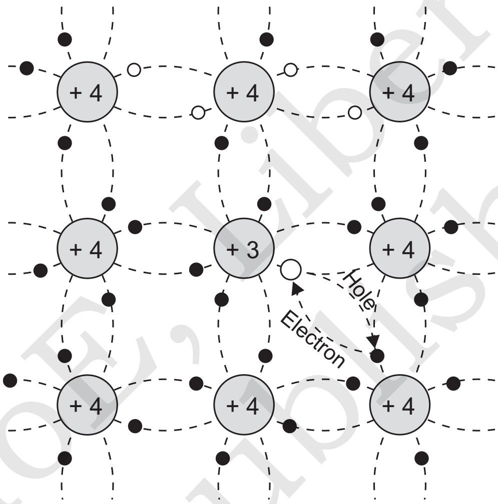


Fig. 4.18. Trivalent acceptor atom (In. Al, B etc.) doped in tetravalent Si or Ge lattice giving *p*-type semiconductor.

In a *p*-type semiconductor, holes are the majority charge carriers and the free electrons are the minority charge carriers.

Like in an *n*-type semiconductor, a *p*-type semiconductor also satisfies the relation

$$pn = p_i \, n_i = n_i^2$$

For each impurity atom, there is a free hole in the valence band but there is no corresponding generation of free electron in the conduction band. So, $p > p_i$ but $n < n_i$.

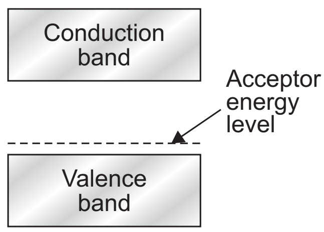


Fig. 4.19. Acceptor energy level.

4.17 DISTINCTION BETWEEN N-TYPE SEMICONDUCTORS AND p-TYPE SEMICONDUCTORS

n-Type Semiconductors	*p-Type Semiconductors*
1. These are extrinsic semiconductors obtained by doping impurity atoms of group V to Ge or Si crystal.	**1.** These are extrinsic semiconductors obtained by doping impurity atoms of group III to Ge or Si crystal.
2. The impurity atoms are called donors. These provide free electrons.	**2.** The impurity atoms are called acceptors. These create vacancies of electrons (or holes).
3. The donor energy level lies just below the conduction band.	**3.** The acceptor energy level lies just above the valence band.
4. The electrons are majority charge carriers and holes are minority charge carriers.	**4.** The holes are majority charge carriers and electrons are minority charge carriers.
5. The free electron density is much greater than hole density *i.e.*, $n_e >> n_h$.	**5.** The hole density is much greater than free electron density *i.e.*, $n_h >> n_e$.
6. The fermi energy level lies in between the donor energy level and the conduction band.	**6.** The fermi energy level lies in between the acceptor energy level and valence band.

4.18 *p-n* JUNCTION

A *p-n* junction is the basic building block of many semiconductor devices like diodes, transistors, etc. A clear understanding of the junction behaviour is important to analyse the working of other semiconductor devices.

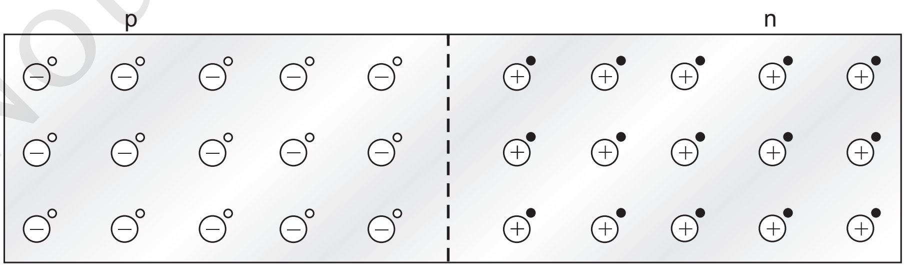


Fig. 4.20. (*a*) *p-n* junction.

Formation of p-n Junction

Consider a thin *p*-type silicon (*p*-Si) semiconductor wafer. By adding precisely a small quantity of pentavalent impurity, part of the *p*-Si wafer can be converted into *n*-Si. The wafer now contains *p*-region and *n*-region and a metallurgical junction between *p*-, and *n*-regions.

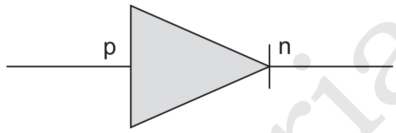


Fig. 4.20. (*b*) Symbol for *p*-*n* junction diode.

Structural Details

p-type and *n*-type semiconductors, taken individually, have practically not much of a use. However, if the two are joined together, they become very useful.

A p-n junction is a single semiconductor crystal that has been selectively doped so that one region is n-type material and the adjacent region is p-type material.

Formation of Depletion Region

When an electron diffuses from $n \rightarrow p$, it leaves behind an ionised donor on *n*-side. This ionised donor (positive charge) is immobile as it is bonded to the surrounding atoms. As the electrons continue to diffuse from $n \rightarrow p$, a layer of positive charge (or positive space-charge region) on *n*-side of the junction is developed.

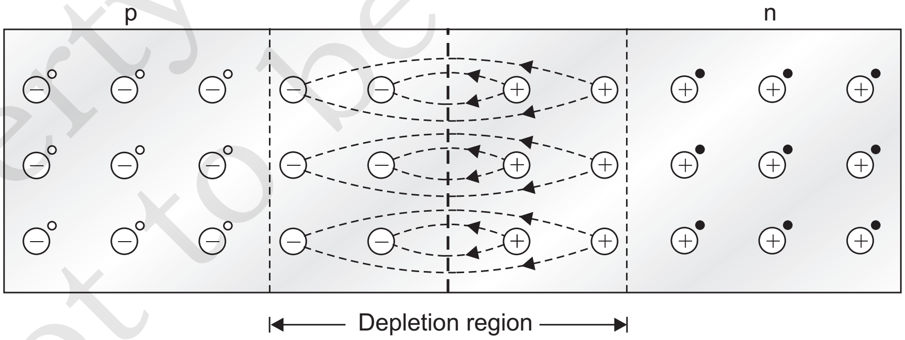


Fig. 4.21. Formation of depletion region in *p*-*n* junction diode.

Similarly, when a hole diffuses from $p \rightarrow n$ due to the concentration gradient, it leaves behind an ionised acceptor (negative charge) which is immobile. As the holes continue to diffuse, a layer of negative charge (or negative space-charge region) on the *p*-side of the junction is developed. This space-charge region on either side of the junction together is known as *depletion region* as the electrons and holes taking part in the initial movement across the junction *depleted* the region of its free charges (Fig. 4.21). The thickness of depletion region is of the order of one-tenth of a micrometre.

The region containing the uncompensated acceptor and donor ions is called depletion region. This is because there is a depletion of mobile charges (holes and free electrons) in the region. Moreover, since this region contains only the immobile ions which are electrically charged, therefore, this region is also called space-charge region.

4.19 SEMICONDUCTOR DIODE (*p-n* JUNCTION DIODE)

Any device which freely allows electric current in one direction but does not allow it in the opposite direction is called a diode. An ideal diode does not allow any current in the reverse direction.

A *p-n* junction acts as a diode. Infact, it may be regarded as an ideal diode because it allows very small current in the opposite direction. A *p-n* junction is generally referred to as "*p-n* junction diode."

A semiconductor diode [Fig. 4.22(*a*)] is basically a *p-n* junction with metallic contacts provided at the ends for the application of an external voltage. It is a two terminal device. A *p-n* junction diode is symbolically represented as shown in Fig. 4.22(*b*).

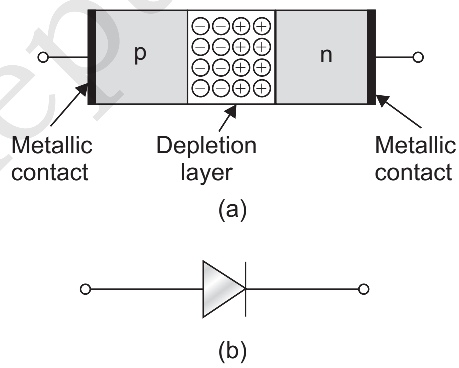


Fig. 4.22. (*a*) Semiconductor diode, (*b*) Symbol for *p-n* junction diode.

4.20 *p-n* JUNCTION DIODE UNDER FORWARD BIAS

When an external voltage V is applied across a semiconductor diode such that p-side is connected to the positive terminal of the battery and n-side to the negative terminal, it is said to be forward biased.

The applied voltage mostly drops across the depletion region and the voltage drop across the *p*-side and *n*-side of the junction is negligible. The direction of the applied voltage (V) is opposite to the built-in potential V_0. As a result, the depletion layer width decreases and the barrier height is reduced [Fig. 4.23]. The effective barrier height under forward bias is $(V_0 - V)$.

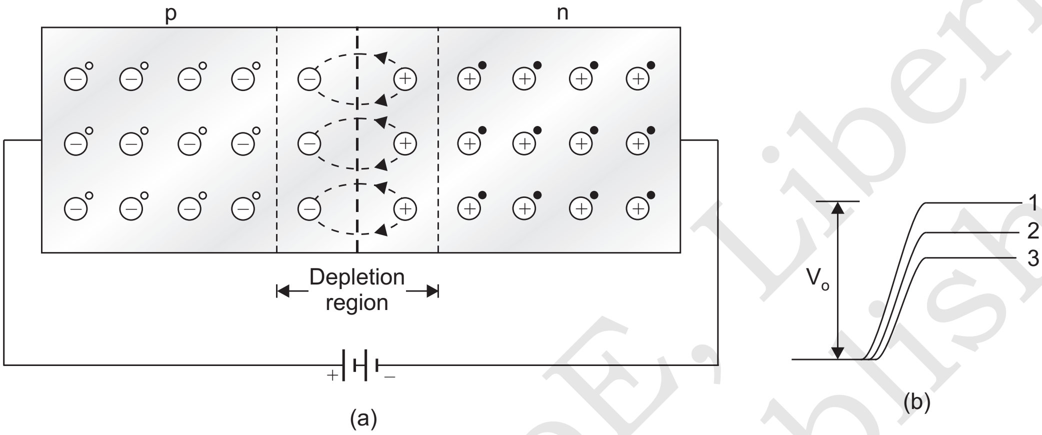


Fig. 4.23. (*a*) *p-n* junction diode under forward bias. (*b*) Barrier potential.

4.21 p-n JUNCTION DIODE UNDER REVERSE BIAS

When an external voltage (V) is applied across the diode such that n-side is positive and p-side is negative, it is said to be reverse biased. The applied voltage mostly drops across the depletion region. The direction of applied voltage is same as the direction of barrier potential. As a result, the barrier height increases and the depletion region widens due to the change in the electric field. The effective barrier height under reverse bias is $(V_0 + V)$, [Fig. 4.24 (*b*)]. This suppresses the flow of electrons from $n \rightarrow p$ and holes from $p \rightarrow n$. Thus, diffusion current decreases enormously compared to the diode under forward bias.

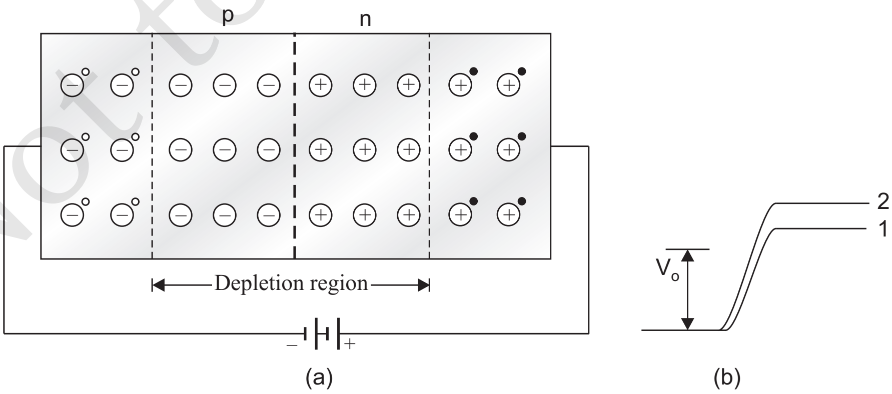


Fig. 4.24. (*a*) Diode under reverse bias. (*b*) Barrier potential under reverse bias.

The electric field direction of the junction is such that if electrons on *p*-side or holes on *n*-side in their random motion come close to the junction, they will be swept to its majority zone. This drift of carriers gives rise to current. The drift current is of the order of a few μA. This is quite low because it is due to the motion of carriers from their minority side to their majority side across the junction. The drift current is also there under forward bias but it is negligible (μA) when compared with current due to injected carriers which is usually in mA.

The diode reverse current is not very much dependent on the applied voltage. Even a small voltage is sufficient to sweep the minority carriers from one side of the junction to the other side of the junction. The current is not limited by the magnitude of the applied voltage but is limited due to the concentration of the minority carriers on either side of the junction.

The current under reverse bias is essentially voltage-independent upto a critical reverse bias voltage, known as breakdown voltage (V_{br}). When $V = V_{br}$, the diode reverse current increases sharply. Even a slight increase in the bias voltage causes large change in the current. If the reverse current is not limited by an external circuit below the rated value (specified by the manufacturer) the *p-n* junction will get destroyed. Once it exceeds the rated value, the diode gets destroyed due to overheating. This can happen even for the diode under forward bias, if the forward current exceeds the rated value.

Example 1: *The V-I characteristic of a silicon diode is shown in the Fig. 4.25. Calculate the resistance of the diode at (a) I_D = 15 mA and (b) V_D = – 10 V.*

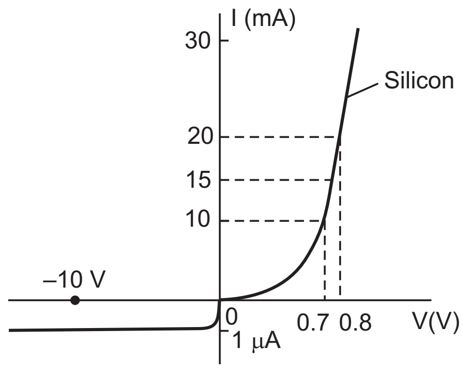


Fig. 4.25

Solution: Considering the diode characteristic as a straight line between I = 10 mA to I = 20 mA, we can calculate the resistance using Ohm's law.

(*a*) From the curve, at I = 20 mA, V = 0.8 V, I = 10 mA, V = 0.7 V,

$r_{fb} = \Delta V/\Delta I$ = 0.1 V/10 mA = **10 Ω**

(*b*) From the curve at V = – 10 V, I = – 1 μA.

Therefore, r_{rb} = 10 V/1 μA = $\mathbf{1.0 \times 10^7}$ Ω

4.22 LIGHT-EMITTING DIODE (LED)

(i) Now-a-days, we can hardly avoid the brightly-coloured "electronic" numbers that glow at us from cash registers and gasoline pumps, microwave ovens and alarm clocks. In nearly all cases, this light is emitted from a *p-n* junction operating as a light-emitting diode (LED).

(ii) ***Light-emitting diode (LED)*** *is a heavily doped p-n junction diode which under forward bias emits visible light.* The light energy is produced by the recombination of electrons and holes at the junction.

Light-emitting diodes are generally made from semiconducting materials gallium arsenide or indium phosphide. Silicon or Germanium diodes emit radiation in the infrared region.

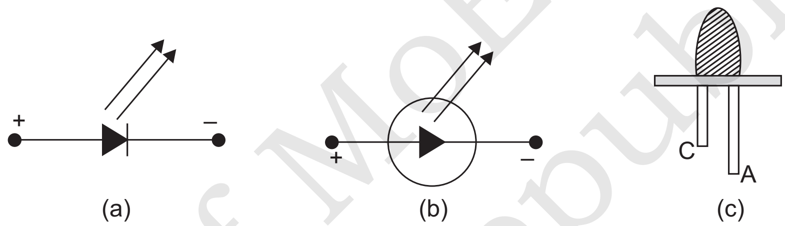


Fig. 4.26

Figs. 4.26(*a*) and (*b*) give the two symbols of the light emitting diode. The actual shape is shown in Fig. 4.26(*c*). The shorter, of its two leads, corresponds to its *n* (or cathode side) while the longer lead corresponds to its *p* (or anode side).

A circuit which uses light-emitting diode is shown in Fig. 4.27. The diode has been forward-biased. The brightness can be controlled by R_L.

If light-emitting diode is reverse-biased, then no light would be emitted at all. In-fact, the LED can be damaged on being reverse-biased.

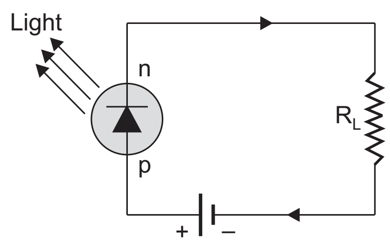


Fig. 4.27. Light-emitting diode.

4.23 PHOTODIODE

A *p-n* junction diode made of photosensitive semiconductor is called **photodiode.** A photodiode is a special purpose *p-n* junction diode fabricated with a transparent window to allow light to fall on the diode. Fig. 4.28 shows the symbolic representation of a photodiode.

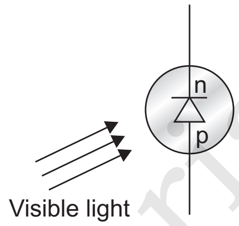


Fig. 4.28. Photodiode.

In a semiconductor, the electrons jump from valence band to the conduction band by absorbing energy from some external source of energy. If the incident visible light is the external source of energy, then the semiconductor is said to be photosensitive. When visible light is incident on a photosensitive semiconductor, more electrons become available to participate in conduction. Thus, the conductivity of a photosensitive semiconductor increases when light is incident on it.

It is generally operated under reverse bias. It is easier to observe the change in the current with change in the light intensity, if a reverse bias is applied. Thus photodiode can be used as a photodetector to detect optical signals. When the photodiode is illuminated with light (photons).

Fig. 4.29 shows an experimental arrangement in which photodiode has been reverse-biased. The applied voltage is less than the breakdown voltage. When the intensity of light increases to a value, say E_0, the current becomes maximum. This maximum current is called saturation current.

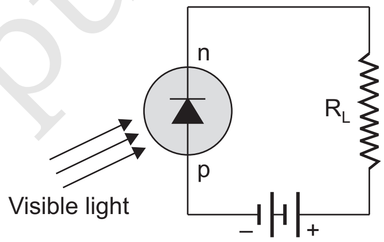


Fig. 4.29. Reverse-biasing of photodiode.

If the photodiode is forward biased as shown in Fig. 4.30, then a certain current exists in the circuit even when no visible light is made incident. This current is called **dark current.** It is represented by OA in the graph of Fig. 4.31.

When visible light of suitable energy is made incident on the photodiode, more electrons move from valence band to conduction band. Consequently, the current increases. The variation of current with intensity of incident light is shown graphically in Fig. 4.31. When the intensity becomes equal to E_0, the current attains its maximum value. It is called saturation current. It is represented by the straight portion BC in the graph of Fig. 4.31.

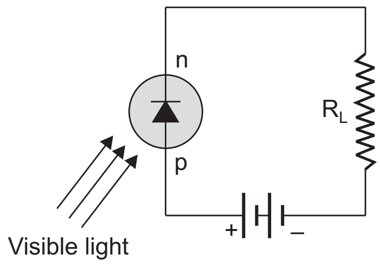


Fig. 4.30. Forward-biasing of photodiode.

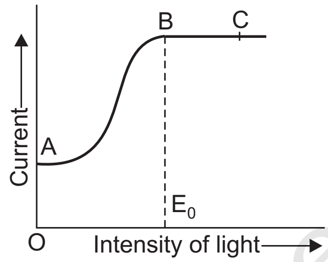


Fig. 4.31. Variation of current with intensity of incident light.

Typical I-V characteristics of a photodiode are shown in Fig. 4.32.

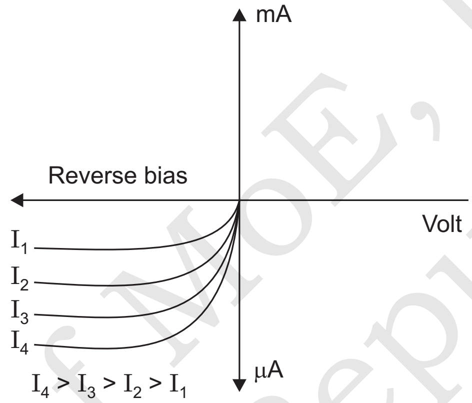


Fig. 4.32. I-V characteristics of a photodiode for different illumination intensities $I_4 > I_3 > I_2 > I_1$.

4.24 JUNCTION TRANSISTOR

By adding another junction to a *p-n* junction diode, we can obtain a device which can control the flow of majority charge carriers. This device is known as **transistor.** It is a combination of two words "transfer" and "resistor".

Transistor was first invented in 1948 by J. Bardeen and W.H. Brattain of Bell Telephone Laboratories, USA. The modern version of transistor was made by W. Shockley in 1951.

Transistor consists of two *p-n* junctions back-to-back. It is obtained by sandwiching either *p*-type or *n*-type semiconductor between a pair of opposite type of semiconductors. While the central slice is called the base, the left and the right crystals are called the emitter and the collector respectively.

All transistors are not alike. Broadly speaking, we have two different types of transistors, namely, *n-p-n* transistor and *p-n-p* transistor.

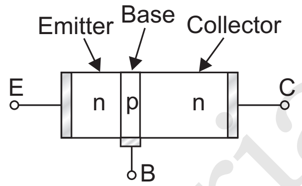


Fig. 4.33. *n-p-n* transistor.

(*i*) **n-p-n transistor** consists of a thin slice of *p*-type semiconductor sandwiched between two much thicker *n*-type semiconductors (Fig. 4.33).

The thin slice of *p*-type semiconductor is called the base. The left hand block of *n*-type semiconductor is called the emitter. The right hand block of *n*-type semiconductor is called the collector.

The symbol of *n-p-n* transistor is shown in Fig. 4.34. The direction of the arrow represents the direction of flow of current. It may be noted that it is opposite to the direction of motion of electrons.

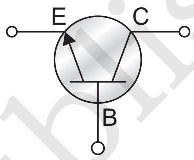


Fig. 4.34. Symbol of *n-p-n* transistor.

(*ii*) **p-n-p transistor** consists of a thin slice of a few microns (10^{-6} m) of *n*-type semiconductor sandwiched between two much thicker *p*-type semiconductors [Fig. 4.35].

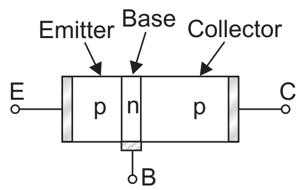


Fig. 4.35. *p-n-p* transistor.

The thin slice of *n*-type semiconductor is called the base. The left hand block of *p*-type semiconductor is called the emitter. The right hand block of *p*-type semiconductor is called the collector.

The symbol of *p-n-p* transistor is shown in Fig. 4.36. The direction of the arrow represents the direction of current. It is the same as the direction of motion of holes. In *p-n-p* transistor, the charge carriers are mainly holes.

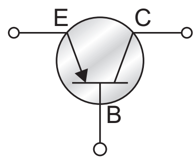


Fig. 4.36. Symbol of *p-n-p* transistor.

4.25 THREE BLOCKS OF A TRANSISTOR

The three blocks of a transistor are not equal. Further, for getting correct transistor action, the doping levels in the different blocks are kept different as explained below.

(i) Emitter. This is the left hand block of the transistor. It is of moderate size and heavily doped semiconductor. This supplies a large number of majority carriers for the current flow through the transistor.

(ii) Base. This is the central block. It is very thin and lightly doped.

(iii) Collector. This collects a major portion of the majority carriers supplied by the emitter. The collector side is moderately doped and larger in size as compared to emitter.

Understanding of Cathode Ray Tube - CRT

Wrorking Principle

When two metal plates are connected to a high voltage source, the negatively charged plate called the cathode emits an invisible ray. The cathode ray is drawn to the positively charged plate, called the anode, where it passes through a hole and continues traveling to the other end of the tube. When the ray strikes the specially coated surface, the cathode ray produces a strong fluorescence or bright light. When an electric field is applied across the cathode ray tube, the cathode ray is attracted by the plate bearing positive charges. Therefore a cathode ray must consist of negatively charged particles. A moving charged body behaves like a tiny magnet, and it can interact with an external magnetic field. The electrons are deflected by the magnetic field. When the external magnetic field is reversed, the beam of electrons is deflected in the opposite direction.

In a cathode ray tube, the cathode is a heated filament and it placed in a vacuum. The ray is a stream of electrons that naturally pour off a heated cathode into the vacuum. Electrons are negative. The anode is positive, so it attracts the electrons pouring off the cathode. In a TV's cathode ray tube, the stream of electrons is focused by a focusing anode into a tight beam and then accelerated by an accelerating anode. This tight, high-speed beam of electrons flies through the vacuum in the tube and hits the flat screen at the other end of the tube. This screen is coated with phosphor, which glows when struck by the beam.

Opperation of CRT

Cathode Ray Tube (CRT) is a computer display screen, used to display the output in a standard composite video signal. The working of CRT depends on the movement of an electron beam which moves back and forth across the back of the screen. The source of the electron beam is the electron gun; the gun is located in the narrow, cylindrical neck at the extreme rear of a CRT which produces a stream of electrons through a thermionic emission. Usually, a CRT has a fluorescent screen to display the output signal. A simple CRT is shown in Fig. 4.37 below.

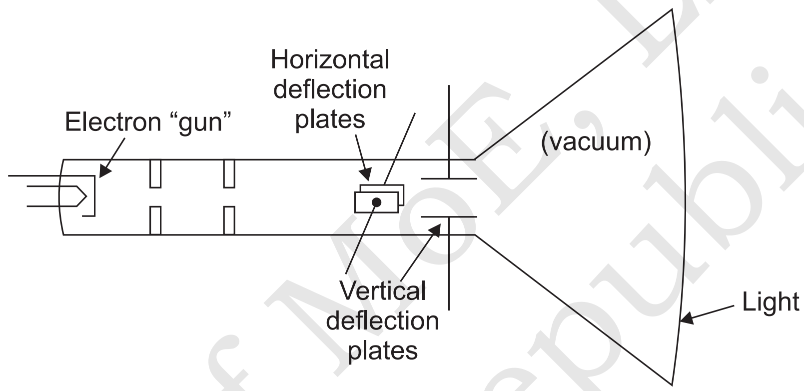


Fig. 4.37. Cathode Ray Tube

The operation of a CRT monitor is very simple. A cathode-ray tube consists of one or more electron guns, possibly internal electrostatic deflection plates, and a phosphor target. CRT has three electron beams one for each (Red, Greern, and Blue) is clearly shown in Fig. 4.37. The electron beam produces a tiny, bright visible spot when it strikes the phosphor-coated screen. In every monitor device, the entire front area of the tube is scanned repetitively and systematically in a fixed pattern called a raster. An image (raster) is displayed by scanning the electron beam across the screen. Thus CRT produces the three colour images which are primary colours.

The main parts of the cathode ray tube are cathode, control grid, deflecting plates and screen.

(*i*) **Cathode**. The heater keeps the cathode at a higher temperature and electrons flow from the heated cathode towards the surface of the cathode. The accelerating anode has a small hole at its center and is maintained at a high potential, which is of positive polarity. The order of this voltage is 1 to 20 kV, relative to the cathode. This potential difference creates an electric field directed from right to left in the region between the accelerating anode and the cathode. Electrons pass through the hole in the anode travel with constant horizontal velocity from the anode to the fluorescent screen, The electrons strike the screen area and it glows brightly.

(*ii*) **The Control Grid.** The control grid regulates the brightness of the spot on the screen. By controlling the number of electrons by the anode and hence the focusing anode ensures that electrons leaving the cathode in slightly different directions are focused down to a narrow beam and all arrive at the same spot on the screen.

(*iii*) **Deflecting Plates.** Two pairs of deflecting plates allow the beam of electrons. An electric field between the first pair of plates deflects the electrons horizontally, and an electric field between the second pair deflects them vertically. The electrons travel in a straight line from the hole in the accelerating anode to the centre of the screen when no deflecting fields are present, where they produce a bright spot.

REVIEW EXERICSE

A. MULTIPLE CHOICE QUESTIONS (MCQs)

1. In a circuit containing a capacitor, an inductor and a resistor in series, V_C , V_L and V_R represent the potential differences across those components and I represents the current through them. Which of the following statements is true ?

1. V_C and I are 180° out of phase.
2. V_R and I are 90° out of phase.
3. V_L and V_C are 180° out of phase.

(*a*) if 1, 2, 3 are correct (*b*) if 1, 2 are correct

(*c*) if 2, 3 are correct (*d*) if 1 only

(*e*) if 3 only.

2. An alternating current of 1.5 mA rms and angular frequency $\omega = 100$ rad s^{-1} flows through a 10 kΩ resistor and a 0.50 μF capacitor in series. The rms potential difference across the capacitor is

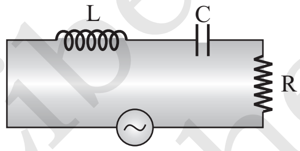
(*a*) 4.8 V (*b*) 15 V (*c*) 30 V (*d*) 34 V

(*e*) 190 V.

3. An LCR series circuit with R = 100 Ω is connected to a 200 V, 50 Hz ac source. When only the capacitance is removed, the current lags the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. The current in the circuit is

(*a*) 2 A (*b*) 1 A (*c*) $\frac{\sqrt{3}}{2}$ A (*d*) $\frac{2}{\sqrt{3}}$ A.

4. Given LCR circuit has L = 5 H, C = 80 μF, R = 40 Ω and variable frequency source of 200 V. What is the source frequency which drives the circuit at resonance ?



(*a*) 25 Hz (*b*) $\frac{25}{\pi}$ Hz (*c*) 50 Hz (*d*) $\frac{50}{\pi}$ Hz.

5. The instantaneous values of current and voltage in an ac circuit are given by

$I = 6 \sin\left(100\,\pi t + \frac{\pi}{4}\right)$, $V = 5 \sin\left(100\,\pi t - \frac{\pi}{4}\right)$. Then

(*a*) current leads the voltage by 45°.
(*b*) voltage leads the current by 90°.
(*c*) current leads the voltage by 90°.
(*d*) voltage leads the current by 45°.

6. A resistor and a capacitor are connected in series with an ac source. If the potential drop across the capacitor is 5 V and that across resistor is 12 V, the applied voltage is

(*a*) 13 V (*b*) 17 V (*c*) 5 V (*d*) 12 V.

7. A capacitor and a coil in series are connected to a 6 volt ac source. By varying the frequency of the source, maximum current of 600 mA is observed. If the same coil is now connected to a cell of emf 6 volt and internal resistance of 2 ohm, the current through it will be

(*a*) 0.5 A (*b*) 0.6 A (*c*) 1.0 A (*d*) 2.0 A.

8. In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the LCR circuit is

(*a*) 242 W (*b*) 305 W (*c*) 210 W (*d*) Zero W.

9. A 50 volt ac is applied across an RC (series) network. The rms voltage across the resistance is 40 volt. Then the potential difference across the capacitance would be

(*a*) 10 V (*b*) 20 V (*c*) 30 V (*d*) 40 V.

10. A *n-p-n* transistor having ac current gain of 50 is to be used to make an amplifier of power gain of 300. What will be the voltage gain of the amplifier ?

(*a*) 8.5 (*b*) 6 (*c*) 4 (*d*) 3.

11. In a semiconducting material, $\left(\frac{1}{5}\right)^{\text{th}}$ of the total current is carried by the holes and the remaining is carried by the electrons. The drift speed of electrons is twice that of holes at this temperature. The ratio between the number densities of electrons and holes is

(*a*) $\frac{21}{6}$ (*b*) 5 (*c*) $\frac{3}{8}$ (*d*) 2.

12. In a common emitter amplifier, the input signal is applied across

(*a*) anywhere (*b*) emitter-collector

(*c*) collector-base (*d*) base-emitter.

13. A common emitter amplifier has a voltage gain of 50, an input impedance of 100 W and an output impedance of 200 W. The power gain of the amplifier is

(*a*) 500 (*b*) 1000 (*c*) 1250 (*d*) 50.

14. The device that can act as a complete electronic circuit is

(*a*) junction diode (*b*) integrated circuit

(*c*) junction transistor (*d*) Zener diode.

15. Which one of the following bonds produces a solid that reflects light in the visible region and whose electrical conductivity decreases with temperature and has high melting point ?

(*a*) metallic bonding (*b*) Van der Waal's bonding

(*c*) ionic bonding (*d*) covalent bonding.

B. FILL IN THE BLANKS

1. A light bulb is rated 100 W for a 220 V supply. The resistance of the bulb and the peak voltage of the source respectively are ....

2. The Q-factor of an LCR circuit in series is largest when ....

3. An LCR series ac circuit is at resonance with 10 V each across L, C and R. If the resistance is halved, the respective voltage across L, C and R are ....
4. An alternating supply of 220 volt is applied across a circuit with resistance 22 ohm and impedance of 44 ohm. The power dissipated in the circuit is ....
5. A fully charged capacitor C with initial charge q_0 is connected to a coil of self-inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is ....
6. When a *p-n* junction is forward biased, the flow of current across the junction is mainly due to ....
7. When a semiconductor is doped with a *p*-type impurity, each impurity atom will ....
8. In *p*-type semiconductor, increase in dopant concentration will ....
9. The collector supply voltage is 6 V and the voltage drop across a resistor of 600 Ω in the collector circuit is 0.6 V, in a transistor connected in common emitter mode. If the current gain is 20, the base current is ....
10. When the voltage across a *p-n* junction diode is increased from 0.65 V to 0.70 V, the change in the diode current is 5 mA. The dynamic resistance of the diode is ....

C. VERY SHORT ANSWER TYPE QUESTIONS

1. What is a logic gate?
2. Write down the truth table of NOR gate and also draw its logic symbol.
3. What is an integrated circuit?
4. Write the truth table of NAND gate.
5. Give the logic symbol of NOR gate.

D. SHORT ANSWER TYPE QUESTIONS

1. How many NAND gates are required to get an AND gate ?
2. How many NAND gates are required to make one NOT gate ?
3. How many NAND gates are required to get an OR gate ?
4. The output of a two-input NAND gate is fed as input to a NOT gate. Write down the truth table for the final output of the combination.
5. What will be the values of inputs A and B for the Boolean equation $(\overline{A + B}) . (\overline{A . B}) = 1$?

E. LONG ANSWER TYPE QUESTIONS

1. Two amplifiers are connected one after the other in series (cascaded). The first amplifier has a voltage gain of 10 and the second has a voltage gain of 20. If the input signal is 0.01 volt, calculate the output ac signal.

2. A *p-n* photodiode is fabricated from a semiconductor with band gap of 2.8 eV. Can it detect a wavelength of 6000 nm?

3. The number of silicon atoms per m^3 is 5×10^{28}. This is doped simultaneously with 5×10^{22} atoms per m^3 of Arsenic and 5×10^{20} per m^3 atoms of Indium. Calculate the number of electrons and holes. Given that $n_i = 1.5 \times 10^{16}\ m^{-3}$. Is the material *n*-type or *p*-type ?

4. In an intrinsic semiconductor, the energy gap E_g is 1.2 eV. Its hole mobility is very much smaller than electron mobility and independent of temperature. What is the ratio between conductivity at 600 K and that at 300 K ? Assume that the temperature dependence of intrinsic carrier concentration n_i is given by

$$n_i = n_0 \exp\left(\frac{-E_g}{2k_BT}\right)$$

where n_0 is a constant.

5. In a *p-n* junction diode, the current I can be expressed as $I = I_0 \left[\exp\left(\frac{eV}{k_BT}\right) - 1\right]$ where I_0 is called the reverse saturation current, V is the voltage across the diode and is positive for forward bias and negative for reverse bias, and I is the current through the diode, k_B is the Botzmann constant (8.6×10^{-5} eV/K) and T is the absolute temperature. If for a given diode $I_0 = 5 \times 10^{-12}$ A and $T = 300$ K, then

(a) what will be the forward current at a forward voltage of 0.6 V ?

(b) what will be the increase in the current if the voltage across the diode is increased to 0.7 V ?

(c) what is the dynamic resistance ?

(d) what will be the current if reverse bias voltage changes from 1 V to 2 V ?

PERIOD-5

TOPIC

5

Atomic and Nuclear Physics

P12CH5

5.1 ATOMIC MASSES

The number of molecules in one mole of carbon is 6.02×10^{23} (Avogadro number). Since carbon is monatomic, therefore, there are 6.02×10^{23} atoms of carbon. These have a mass of 12 g.

$$\therefore \quad \text{Mass of 1 atom of carbon} = \frac{12}{6.02 \times 10^{23}} \text{ g} = \frac{12}{6.02 \times 10^{26}} \text{ kg}$$

$$\therefore \quad 1 \text{ amu} = \frac{1}{12} \times \frac{12}{6.02 \times 10^{26}} \text{ kg} = 1.66 \times 10^{-27} \text{ kg}$$

Atomic masses are conventionally expressed in atomic mass units such that the mass of the most abundant type of carbon atom is, by definition, exactly 12.00 atomic mass unit.

5.2 COMPOSITION OF NUCLEUS—PROTONS AND NEUTRONS

Rutherford's experiment on the scattering of α-rays led us to conclude that an atom has a tiny central hard core. Nearly the whole mass of the atom is concentrated in this core. This central core is named as *nucleus*. The electrons revolve around the nucleus in different orbits in the same manner in which planets of our solar system revolve around the Sun. It is for this reason that electrons are sometimes called *planetary electrons*.

Every atomic nucleus contains basically two types of particles-protons and neutrons. The single exception is that of nucleus of hydrogen. This nucleus contains only one proton.

The total number of protons in atomic nucleus is equal to total number of electrons in atom. So, the total amount of negative charge present in atom is equal to the total amount of positive charge in the atom. This makes the atom, as a whole, electrically neutral.

A nucleus provides distinct individuality to an atom. Atoms of different elements possess different atomic nuclei. A nucleus has no well-defined boundary. However, for the sake of convenience, the nucleus is regarded as a hard sphere. It has been estimated that the size of the nucleus is of the order of 10^{-14} m. When compared with the size of atom (10^{-10} m), we conclude that an atom has a lot of empty space in it. A nucleus is symbolically represented as ${}^{A}_{Z}X$. Thus, Uranium nucleus is denoted by ${}^{235}_{92}U$. This indicates that there are 92 protons in Uranium nucleus and (235 – 92) *i.e.,* 143 neutrons. In other words, the mass number of Uranium nucleus is 235 and atomic number is 92. The mass number of a nucleus is the total number of nucleons in the nucleus. This determines the mass of the nucleus.

The number of neutrons in a nucleus is given by,

$$N = A - Z$$

where A is mass number and Z is atomic number of nucleus.

Proton is a fundamental particle. It may also be called the *nucleus of hydrogen.* It has positive charge of 1.6×10^{-19} C. Its mass is 1.67×10^{-27} kg. It is 1836 times heavier than an electron.

Neutron is also a fundamental particle. It is an integral constituent of all nuclei except that of hydrogen. It is an electrically neutral entity which was discovered in 1932. Its mass is 1.675×10^{-27} kg. So, it is 1840 times heavier than an electron.

5.3 SIZE OF NUCLEUS

It has been confirmed by various experiments that the nucleus does not have a sharp or well-defined boundary.

However, the nuclear radius R can be given by $R = R_0 A^{1/3}$, where R_0 (= 1.2×10^{-15} m) is a constant which is the same for all nuclei and A is the mass number of the nucleus.

The unit of nuclear radius is fermi of the order of 1 fm = 10^{-15} m

The nuclear radii range from 1 fm to 10 fm.

Nuclear volume, $$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

It is clear from here that the nuclear volume is proportional to mass number.

---

*This is, roughly speaking, atomic weight.

**Nucleons are the particles present in the nucleus.

5.4 NUCLEAR DENSITY

Consider a nucleus of mass number A and radius R.

$$\text{Mass of nucleus} = \text{A amu} = A \times 1.66 \times 10^{-27} \text{ kg}$$

$$\text{Volume of nucleus} = \frac{4}{3}\pi R_0{}^3 A = \frac{4}{3} \times \frac{22}{7} \times (1.2 \times 10^{-15})^3 \text{ A m}^3$$

$$= 7.24 \times 10^{-45} \text{ A m}^3$$

$$\text{Density of nucleus,} \quad \rho = \frac{A \times 1.66 \times 10^{-27}}{7.24 \times 10^{-45} \times A} \text{ kg m}^{-3}$$

$$= 2.29 \times 10^{17} \text{ kg m}^{-3}$$

Discussion. (i) The nuclear density does not depend upon mass number. So, we can safely conclude that all nuclei possess nearly the same density.

(ii) The nuclear density has extremely large value. Such high densities are found in white dwarf stars which contain mainly the nuclear matter.

(iii) The nuclear density is *not uniform* throughout the nucleus. It has maximum value at the centre and decreases gradually as we move away from the centre of the nucleus.

Example 2: *Find the density of nuclear mass in* ${}^{238}_{92}U$ *nucleus. Given :* R_0 *= 1.5 fermi, mass of each nucleon =* 1.67×10^{-27} *kg.*

Solution: Mass number of ${}^{238}_{92}U = 238$

$$R_0 = 1.5 \text{ fm} = 1.5 \times 10^{-15} \text{ m}$$

Mass of each nucleon

$$= 1.67 \times 10^{-27} \text{ kg}$$

$$\text{Density,} \quad \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{1.67 \times 10^{-27} \times A}{\frac{4}{3}\pi R^3}$$

$$= \frac{1.67 \times 10^{-27} \times A \times 3 \times 7}{4 \times 22\,(R_0 A^{1/3})^3}$$

or

$$\rho = \frac{1.67 \times 10^{-27} \times 21 \times A}{88 \times (1.5 \times 10^{-15})^3 \text{ A}} \text{ kg m}^{-3}$$

$$= \mathbf{1.18 \times 10^{17} \text{ kg m}^{-3}}$$

5.5 ISOTOPES

These are the atoms of the same element having the same atomic number but different atomic weights. Such nuclides having same number of protons but different number of neutrons are called *isotopes*. In other words, they have same atomic number Z, but different mass number A.

The number of orbital electrons is the same in isotopes of elements. This explains as to why they possess identical chemical properties.

The isotopes of an element may have same name or different names. As an example, the three isotopes $_1^1H$, $_1^2H$, and $_1^3H$ of hydrogen are known as Hydrogen, Deuterium and Tritium.

5.6 ISOBARS

In Greek language, 'bar' means weight and 'iso' means equal. Two elements which are chemically different, but have physically the same mass are called *isobars*. So, isobars are atoms of different elements having the same atomic mass but different atomic number. Since Z number of isobars are different therefore they do not occupy the same place in periodic table. Also, for the same reason, their chemical properties are widely different from each other.

Examples of isobars. (i) $_{18}^{40}Ar$ and $_{20}^{40}Ca$ **(ii)** $_{26}^{58}Fe$ and $_{27}^{58}Ni$.

5.7 MASS DEFECT

Consider a nucleus of mass number A and atomic number Z. It will contain Z protons and (A – Z) neutrons. The mass of the constituent nucleons will be $[Zm_p + (A - Z)m_n]$. Here m_p and m_n are the masses of proton and neutron respectively.

It has been observed that the rest mass M of the nucleus is always less than the mass of the constituent particles. *The difference between the rest mass of the nucleus and the sum of the masses of the nucleons composing a nucleus is known as* ***mass defect.*** *It is given by,*

$$\Delta m = [Zm_p + (A - Z)m_n] - M$$

As an example, consider the mass defect in the case of deuteron. It is an isotope of hydrogen. It contains one proton and one neutron.

m_p = mass of proton = 1.007825 amu

m_n = mass of neutron = 1.008665 amu

$m_p + m_n$ = 2.016490 amu

M = mass of deuteron = 2.014103 amu

Mass defect, $\Delta m = m_p + m_n - M = 0.002387$ amu

Thus, when a proton and neutron are brought together to form a deuteron, a part of their masses is lost.

5.8 BINDING ENERGY

It is the energy required to break up a nucleus into its constituent parts and place them at an infinite distance from one another.

The binding energy is related to mass defect by Einstein's mass-energy relation. If Δm be the mass defect, then

Binding energy = Δmc^2, where c is the speed of light.

In the case of deuteron:

Binding energy = 0.002387 × 931 MeV = **2.22 MeV**

Thus, an energy of 2.22 MeV is required to separate by an infinite distance a neutron from proton. This has been confirmed experimentally.

5.9 UNITS OF ENERGY

The nuclear energy is generally measured in electron volt. It is defined as the amount of energy acquired by an electron when accelerated through a potential difference of 1 volt.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V} = 1.6 \times 10^{-19} \text{ J}$$

The megaelectron volt (MeV) is a larger energy and is defined as 1 million eV.

So, $1 \text{ MeV} = 1.6 \times 10^{-13}$ J

If another unit of energy is needed, then one may use a unit of mass, since mass and energy are interchangeable. The atomic mass unit is defined as 1/12th of the mass of the carbon atom ${}^{12}_{6}C$. Now the number of molecules in 1 mole of carbon is 6.02×10^{23} (Avogadro constant) and since carbon is monatomic, therefore, there are 6.02×10^{23} atoms of carbon. These have a mass 12 g.

$\therefore$ Mass of 1 atom of carbon = $\dfrac{12}{6.02 \times 10^{23}}$ g = $\dfrac{12}{6.02 \times 10^{26}}$ kg

$\therefore$ 1 amu = $\dfrac{12}{12 \times 6.02 \times 10^{26}}$ kg = 1.66×10^{-27} kg (approx.)

1 kg change in mass produces 9×10^{16} joule.

Again, $E = mc^2 = 1.66 \times 10^{-27} \times 9 \times 10^{16}$ joule

$$= \frac{1.66 \times 10^{-27} \times 9 \times 10^{16}}{1.6 \times 10^{-13}} \text{ MeV} \quad (\because \ 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J})$$

= 931 MeV (approx.)

$\therefore$ 1 amu = 931 MeV (approx.)

5.10 RADIOACTIVITY

Henry Becquerel discovered the phenomenon of radioactivity in 1896. He found that certain compounds of uranium emitted invisible radiations which affected photographic plates. Later on, Thorium and its compounds were also found to behave in a similar way. Piere Curie and Madame Curie discovered a new element, called radium, which showed these properties. They also discovered another similar element which they named as 'polonium'. All these elements possess the property of emitting certain rays, spontaneously of their own accord. This phenomenon of emitting radiations spontaneously is called **radioactivity,** and the substance, which does so, is called a radioactive material.

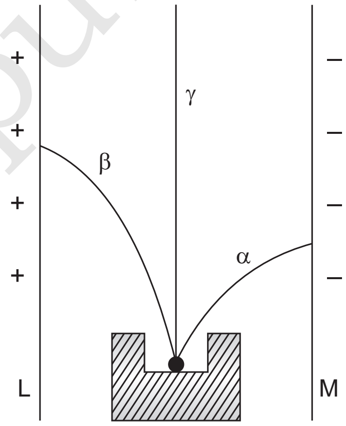


Fig. 5.1. Experimental set-up to demonstrate that there are three types of radiations.

The radiations emitted by a radioactive body are not homogeneous but consist of three distinct types of radiations. These are named as α, β and γ-rays. Also all the radiations are not emitted simultaneously. The nucleus emits a radiation (α or β) and changes into a new nucleus (hence a new element). To balance the energy, γ-radiations are emitted.

To show that there are three types of radiations, let us have the experimental set-up shown in Fig. 5.5. In a thick block of lead with a small hole in its centre, we place a radioactive material. The rays issue out upwards. In the region above, we create a strong electric field by placing two parallel plates connected to positive and negative terminals of a battery. We see that α-rays are deflected towards negative plate and β-rays towards positive plate. This shows that α-rays are positively charged particles. γ-rays go out straight, undeflected. These rays are like light. α and β-rays can be similarly deflected by a suitable magnetic field.

5.11 PROPERTIES OF α-RAYS

1. These are positively charged particles, each having a mass equal to four times the mass of hydrogen atom and charge + $2e$ (twice the charge of an electron). **2.** These are deflected by electric and magnetic fields. **3.** These affect photographic plates and cause fluorescence. **4.** These produce strong ionisation. **5.** These have small penetrating power. A piece of paper or a thin aluminium foil can stop them. **6.** These move with comparatively small velocities. These are nuclei of helium atoms ; and an α-particle consists of a closely packed group of two protons and two neutrons.

5.12 PROPERTIES OF β-RAYS

1. These are negatively charged particles, each having a mass and a charge equal to that of an electron. **2.** These are deflected by electric and magnetic fields. **3.** These affect photographic plates and cause fluorescence in various materials. **4.** These produce less ionisation than α-particles. **5.** These have a large penetrating power, greater than that of α-particles. An aluminium sheet 1 cm thick can stop β-rays. **6.** These move with greater velocities, sometimes up to 90% of the velocity of light. **7.** The mass of a β-particle increases with its velocity according to Einstein's relativistic mass relation.

5.13 PROPERTIES OF γ-RAYS

1. These are electromagnetic waves like light or X-rays, only having much greater energy, $h\nu$, and much smaller wavelength λ, than X-rays. **2.** These are undeflected by electric or magnetic fields. **3.** These produce little or no ionisation. **4.** These have the strongest penetrating power. These can pass through many centimetre of lead. **5.** These affect photographic plates and cause fluorescence similar to X-rays. **6.** These travel with the velocity of light. **7.** These are very harmful to human tissues. **8.** These produce photoelectric effect when these are made incident on solid surfaces.

5.14 SODDY'S DISPLACEMENT LAW OF RADIOACTIVE TRANSFORMATIONS

1. When a nucleus ejects an α-particle, the mass becomes less by 4 units and charge decreases by 2 units. Thus, the nucleus ${}^{A}_{Z}Y$ on emission of α-particle gets transformed into a new nucleus ${}^{A-4}_{Z-2}Y$.

$$\therefore \qquad {}^{A}_{Z}Y \xrightarrow{\alpha} {}^{A-4}_{Z-2}Y$$

Thus, the substance shifts or is displaced from its original position in the periodic table, two steps backwards.

2. When a nucleus ejects a β-particle, the mass remains unchanged and the charge increases by 1 unit. So a material ${}^{A}_{Z}Y$ on emission of β-particle gets transformed into a new nucleus as ${}^{A}_{Z+1}Y$.

Thus, the original substance shifts or is displaced one step higher in the periodic table.

$$\text{Thus,} \qquad {}^{A}_{Z}Y \xrightarrow{\beta} {}^{A}_{Z+1}Y$$

3. When a nucleus emits γ-rays, the mass or the charge or the position of the nucleus in the periodic table are not affected. Only some energy is radiated and the original nucleus shifts from higher energy level to lower energy level.

5.15 RUTHERFORD AND SODDY'S LAWS OF RADIOACTIVE DECAY

1. The disintegration of radioactive material is purely a random process and it is merely a matter of chance, which nucleus will suffer disintegration, or decay first.

2. The rate of decay is completely independent of the physical composition and chemical condition of the material.

3. The rate of decay is directly proportional to the quantity of material actually present at that instant. Thus, as the decay goes on, the original material goes on decreasing in quantity and the rate of decay consequently goes on decreasing.

Thus from the third law, if N is the number of radioactive atoms present at any instant, then the rate of decay,

$$-\frac{dN}{dt} \propto N \quad \text{or} \quad -\frac{dN}{dt} = \lambda N,$$

where λ is the decay constant or the disintegration constant.

$$\therefore \qquad \frac{dN}{dt} = -\lambda N$$

On rearranging, $$\frac{dN}{N} = -\lambda\, dt$$

On integration $$\log_e N = -\lambda t + C$$

where C is the integration constant.

If at $t = 0$, we had N_0 atoms, $\log_e N_0 = 0 + C$

Thus, we get $$\log_e N - \log_e N_0 = -\lambda t$$

or $$\log_e \frac{N}{N_0} = -\lambda t$$

or $$\frac{N}{N_0} = e^{-\lambda t} \quad \text{or} \quad \boxed{N = N_0\, e^{-\lambda t}.}$$

This equation represents the radioactive decay law. It gives the number of active nuclei left after time t.

5.16 RADIOACTIVE DISINTEGRATION CONSTANT λ

According to the laws of radioactive decay, we have

$$\frac{dN}{N} = -\lambda\, dt$$

If $dt = 1$ second, then $$\frac{dN}{N} = -\lambda$$

Thus, λ may be defined as the *relative number of atoms decaying per second.*

Again, since $N = N_0\, e^{-\lambda t}$

and if $$t = \frac{1}{\lambda}, \text{ we get } N = N_0\, e^{-1} = \frac{N_0}{e}$$

Thus, λ is also defined as *the reciprocal of the time when* $\frac{N}{N_0}$ *falls to* $\frac{1}{e}$.

5.17 HALF-LIFE PERIOD

Consider the situation when the decaying material is reduced to exactly $\frac{1}{2}$ of its original quantity. The time taken for this decay $\left(\frac{N}{N_0} = \frac{1}{2}\right)$ is called the half-life period of the material. *It is defined as the time required for the disappearance of half of the amount of the radioactive substance originally present.*

If T represents the half-life period, then

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda T} \text{ or } e^{\lambda T} = 2$$

$$\therefore \quad \lambda T = \log_e 2 = 0.6931$$

$$\therefore \quad T = \frac{0.6931}{\lambda} \quad \text{or} \quad \boxed{\lambda = \frac{0.6931}{T}}$$

Combining these relations, we obtain

$$\frac{N}{N_0} = e^{-\lambda t}$$

or $$\frac{N_0}{N} = e^{\lambda t}$$

$$\therefore \quad \log_e \frac{N_0}{N} = \lambda t$$

or $$2.303 \log_{10} \frac{N_0}{N} = \frac{0.6931}{T} t$$

or $$t = \frac{2.303}{0.6931} T \log_{10} \frac{N_0}{N}$$

or $$\boxed{t = 3.323 \, T \log_{10} \frac{N_0}{N}}$$

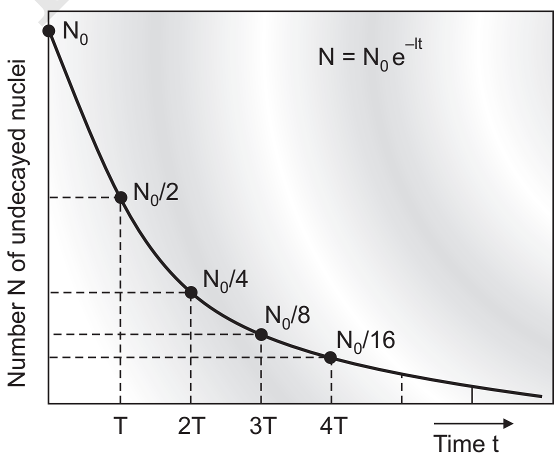


Fig. 5.2. Exponential decay of a radioactive species. After a lapse of T, population of the given species drops by a factor of 2.

This relation shows that a material with a half-life period T changes in quantity from N_0 to N in time t.

5.18 UNITS OF RADIOACTIVITY

The activity of a radioactive sample is generally expressed in terms of its rate of decay. In other words, the activity of a radioactive sample is expressed in terms of the number of disintegrations per unit time. The radioactivity is measured in the following three units.

(i) The curie (Ci). This was originally defined as the activity of 1 g of radium in equilibrium with its by-products. But it is now defined as under :

The activity of a radioactive substance is said to be one curie if it undergoes 3.7 × 10^{10} disintegrations per second.

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ disintegrations/s}$$

Smaller units are millicurie and microcurie.

$$1 \text{ millicurie} = 3.7 \times 10^{7} \text{ disintegrations/s}$$

$$1 \text{ microcurie} = 3.7 \times 10^{4} \text{ disintegrations/s}$$

(ii) The rutherford (Rd). *The activity of a radioactive substance is said to be one rutherford if it undergoes 10^6 disintegrations per second.*

$$1 \text{ rutherford} = 10^{6} \text{ disintegrations s}^{-1}$$

Smaller units are millirutherford and microrutherford.

$$1 \text{ millirutherford} = 10^{3} \text{ disintegrations s}^{-1}$$

$$1 \text{ microrutherford} = 1 \text{ disintegration s}^{-1}$$

(iii) The becquerel (Bq). *It is the SI unit for activity. The activity of a radioactive substance is said to be one becquerel if it undergoes 1 disintegration per second.*

$$1 \text{ becquerel} = 1 \text{ disintegration s}^{-1}$$

Relation between different units

$$1 \text{ curie} = 3.7 \times 10^{4} \text{ rutherford}$$

$$= 3.7 \times 10^{10} \text{ becquerel.}$$

5.19 NUCLEAR REACTIONS

When a nucleus is bombarded with nucleons or other sub-atomic particles, it undergoes a change in composition. A nuclear reaction indicates that change. ***A nuclear reaction may be defined as the***

transformation in nuclei brought about by their interaction with elementary particles or with different nuclei themselves.

Most nuclear reactions involve a nucleus A and a particle '*a*', This pair is known as *parent pair*. After a collision between these two, a new nucleus B is formed and another particle '*b*' is ejected. This pair is called the *final pair*. The nuclear reaction may be expressed as under :

$$A + a \longrightarrow B + b$$

In some reactions, energy Q is evolved. Such a reaction is known as **exothermic or exoergic reaction**. In a reaction in which energy is absorbed, the reaction is known as **endothermic or endoergic reaction**. So, in the final analysis, a nuclear reaction may be written as under :

$$A + a \longrightarrow B + b + Q$$

Here, in the usual expression, *a* is the bullet fired on a target A. This results in the recoil nucleus B and giving the product particle *b* with a release or absorption of reaction energy Q. Q is known as the reaction energy or Q-value of nuclear raction. *The absorption or evolution of energy in a nuclear reaction takes place in accordance with Einstein's mass-energy equivalence relation.*

5.20 CONSERVATION LAWS IN NUCLEAR REACTIONS

Broadly, the following conservation laws are obeyed in nuclear reactions.

(i) Conservation of number of nucleons.

(ii) Conservation of charge. In a reaction, the total electric charge is conserved. This ultimately means that the total Z number, the atomic number is conserved.

(iii) Conservation of linear momentum. Like all physical processes involving collisions, the total momentum along any direction, before and after the event, is always conserved.

(iv) Conservation of angular momentum.

(v) Conservation of mass-energy. According to mass-energy equivalence in the theory of relativity, mass and energy are equivalent. So the principle of conservation of energy in mechanics has to be extended to the conservation of mass-energy in nuclear reactions. The mass-energy equation for the nuclear reaction may be written as :

$$m_1c^2 + E_{k_1} + m_2c^2 + E_{k_2} = m_3c^2 + E_{k_3} + m_4c^2 + E_{k_4}$$

where m_1, m_2, m_3 and m_4 are the rest masses and $E_{k_1}, E_{k_2}, E_{k_3}$ and E_{k_4} are their respective kinetic energies.

5.21 NUCLEAR FISSION

In 1939, German Scientists Otto Hahn and Strassmann while studying nuclear reactions, discovered that when a uranium nucleus is bombarded with a neutron, it explodes into two nearly equal fragments, Barium and Krypton. Since this process somewhat resembles fission of cells in biology, therefore this phenomenon of nuclear disintegration was also called fission.

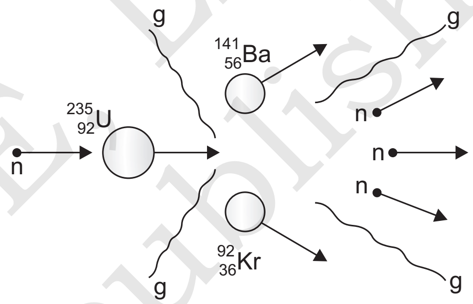


Fig. 5.3. Nuclear fission.

Nuclear fission *is defined as a type of nuclear disintegration in which a heavy nucleus splits up into two nuclei of nearly comparable masses with liberation of energy.*

The fission is accompanied by the release of three neutrons and radiation energy in the form of γ-rays. The reaction is represented as :

$${}^1_0n + {}^{235}_{92}\text{U} \longrightarrow [{}^{236}_{92}\text{U}] \longrightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3{}^1_0n + \gamma$$

The diagrammatic sketch is given in Fig. 5.7. A neutron strikes the ^{235}U nucleus and in the process two nuclides ^{141}Ba and ^{92}Kr are formed with the release of 3 neutrons as shown. The wavy lines indicate the energy released in the form of γ-radiations. An important point to note here is that a *slow* neutron is used to cause fission. Further whereas one neutron is lost in the process to produce fission, three neutrons are produced as a product of the fission. This fact has a tremendous significance in the construction of nuclear bomb.

Energy released in fission. The fission fragments Barium, Krypton and neutrons are released with high velocities. Also energy is released in the form of γ-rays. An estimate can be made as in the example given below :

Before the reaction:

$$\text{Mass of } {}^{235}_{92}\text{U} = 235.0439 \text{ amu} ; \text{Mass of } {}^{1}_{0}n = 1.0087 \text{ amu}$$

$$\text{Total mass} = 236.0526 \text{ amu} \quad ...(i)$$

After the reaction:

$$\text{Mass of } {}^{141}_{56}\text{Ba} = 140.9129 \text{ amu} ; \text{Mass of } {}^{92}_{36}\text{Kr} = 91.8973 \text{ amu}$$

$$\text{Mass of three } {}^{1}_{0}n = 3.0261 \text{ amu}$$

$$\text{Total mass} = 235.8363 \text{ amu} \quad ...(ii)$$

$$\text{Mass defect} = 0.2163 \text{ amu} \quad [(i) - (ii)]$$

Since 1 amu = 931 MeV,

∴ The energy released = 931 × 0.2163 = 201.37 MeV ≈ 200 MeV

This is a huge figure. Calculations reveal that 235 g of Uranium, on complete fission, releases energy equivalent to the burning of about 600 tonnes of coal. However, this 200 MeV consists of K.E. of fission fragments, of released neutrons and of the γ-rays. Eventually, it is transferred to the surrounding matter appearing as heat.

5.22 NUCLEAR FUSION

We know 'fission' to be a process in which a heavy nucleus breaks up into two lighter nuclei. Fusion, on other hand, is the reverse of fission. Thus, fusion *is a process in which lighter nuclei merge into one another to form a heavier nucleus.* As in fission, fusion also is accompanied by a release of energy.

The binding energy per nucleus thus formed is greater than the binding energy per nucleon of the lighter elements, which fuse to form the single nucleus. Taking an example, let us consider the fusion of the deuterium nuclei to form a single helium nucleus :

We know that mass of a deuteron = 2.01471 amu

∴ Mass of two deuterons = 4.02942 amu

Mass of α-particle (*i.e.*, a Helium nucleus) = 4.00388 amu

$\therefore$ Δm, mass defect = 0.02554 amu

Since 1 amu = 931 MeV,

$\therefore$ The energy liberated = 0.02554 × 931 MeV

= 23.78 MeV ≈ 24 MeV

Thus, a single helium nucleus formed out of fusion of two deuterons (*i.e.*, deuterium nuclei) releases 24 MeV energy. In case a large number of helium nuclei are fused, we readily see that a tremendous amount of energy is released.

Since both the deuterons are similarly charged (+ 1), therefore, we require a large amount of energy to bring the two together for fusion against Coulomb repulsion. Though theoretically this energy may be given to them by accelerating them through strong electric field, practically it is not easy or convenient. The other alternative is to give them high thermal energies. In the Sun and the stars, such high temperatures (≈ 10 Million K) are available which impart enough energy to the fusing particles which are protons or deuterons. Thus, such a fusion process is called a *thermonuclear fusion.*

5.23 ENERGY SOURCE OF STARS AND SUN

Proton-Proton Cycle. The interior of Sun is at about 27 million K. The thermonuclear reactions taking place are as follows :

$$ {}_1^1\text{H} + {}_1^1\text{H} \longrightarrow {}_1^2\text{H} + {}_1^0e + 0.4 \text{ MeV} \quad \ldots(i)$$

$$ {}_1^1\text{H} + {}_1^2\text{H} \longrightarrow {}_2^3\text{He} + 5.5 \text{ MeV} \quad \ldots(ii)$$

$$ {}_2^3\text{He} + {}_2^3\text{He} \longrightarrow {}_2^4\text{He} + 2{}_1^1\text{H} + 12.9 \text{ MeV} \quad \ldots(iii)$$

The reactions (*i*) and (*ii*) occur twice. So, the net reaction is :

$$ 4{}_1^1\text{H} \longrightarrow {}_2^4\text{He} + 2{}_1^0\text{e} + 24.7 \text{ MeV}$$

Thus at that high temperature available in the core of the Sun, four protons fuse into a Helium nucleus with the release of two positrons and 24.7 MeV of energy.

Example 4: *A neutron is absorbed by a* ${}_3^6Li$ *nucleus with the subsequent emission of an alpha particle.*

(i) Write the corresponding nuclear reaction.

(ii) Calculate the energy released, in MeV, in this reaction.

Given : mass $^{6}_{3}Li$ = 6.015126 u ; mass (neutron) = 1.0086654 u ; Mass (alpha particle) = 4.0026044 u and mass (triton) = 3.0100000 u. Take 1 u = 931 MeV/c^2.

Solution: $^{6}_{3}\text{Li} + ^{1}_{0}n \longrightarrow ^{4}_{2}\text{He} + ^{3}_{1}\text{H} + Q$

$$\begin{aligned}\text{Total initial mass} &= 6.015126 + 1.0086654 \\ &= 7.0237914 \text{ amu} \\ \text{Total final mass} &= 4.0026044 + 3.01 \\ &= 7.0126044 \text{ amu} \\ \text{Mass defect, } \Delta m &= 7.0237914 - 7.0126044 \\ &= 0.0111870 \text{ amu} \\ \text{Energy released, Q} &= 0.0111870 \times 931 \text{ MeV} \\ &= \mathbf{10.415\ MeV}.\end{aligned}$$

Example 6: *How many α and β-particles are emitted when $^{238}_{92}U$ changes into $^{206}_{82}Pb$ through a series of radioactive decays ?*

or

How many α and β-particles are lost when $^{238}_{92}U$ changes into $^{206}_{82}Pb$?

Solution: Since the emission of β-particle has no effect on mass number, therefore, the change of mass number is purely due to the emission of α-particles. The emission of one α-particle reduces the mass number by 4.

$$\therefore \quad \text{Number of } \alpha\text{-particles emitted} = \frac{238-206}{4} = \frac{32}{4} = \mathbf{8}$$

Due to the emission of 8 α-particles, the charge number reduces by 8 × 2 *i.e.,* 16. So, due to the emission of 8 α-particles, the charge number becomes (92 – 16) *i.e.,* 76. But the charge number of the end product is 82. Clearly, the number of β-particles emitted is (82 – 76) *i.e.,* **6.**

Example 8: *If 200 MeV energy is released in the fission of a sample nucleus of $^{235}_{92}U$, how many fissions must occur per second to produce a power of 1 kW ?*

Solution: Energy released by the fission of one nucleus of $^{235}_{92}U$

$$= 200 \text{ MeV} = 200 \times 1.6 \times 10^{-13} \text{ J}$$

$$= 3.2 \times 10^{-11} \text{ J}$$

$$1 \text{ kW} = 1000 \text{ W} = 1000 \text{ J s}^{-1}$$

If x be the number of fissions per second required to generate a power of 1 kW, then

$$x \times 3.2 \times 10^{-11} = 1000$$

or

$$x = \frac{1000}{3.2 \times 10^{-11}} = \mathbf{3.125 \times 10^{13}}$$

5.24 RADIATION HAZARDS

After the invention of the nuclear reactions as a pure scientific study, man developed the atomic and hydrogen bombs in an attempt to gain supremacy over other men. But in this process, the large amount of radiated energy to which mankind as a whole is exposed is really posing a problem even for human existence. These radiations are causing great dangers to human organism.

For instance, let us see how γ-ray acts on a human system. When γ-ray or any high energy nuclear particle passes through any material, it knocks out electrons from its atoms and ionises them. With the atom thus broken and ionised, the complex molecular structure of the organism becomes weak and may break up. This breaking up of the molecules disrupts the entire normal functioning of the biological system. This leads to a permanent damage of the tissues, and ultimately leads to death.

The extent to which a human organism is damaged depends upon (*i*) the dose and the rate at which the radiation is being given, (*ii*) the part of the body exposed to it. Our hands and feet, not being vital organs, can receive much greater dose than other parts of the body. The damage itself can be either (*i*) **pathological** or (*ii*) **genetic.**

In the **pathological damage,** the organism exposed to the radiation may ultimately die. This happens when the body is exposed to about 600 *r*. Smaller dose of 100 *r* approximately may cause a start of leukemia (death of red blood corpuscles in the blood) or cancer, which on spreading causes death ultimately.

The **genetic damage** is still worse. The radiations cause injury to genes in the reproductive cells. This gives rise to mutations which pass on from generation to generation. Mutations are always harmful and are irreversible. There is no way to escape from the results of this damage. In a simple language, it may mean that a person exposed to such damage may have a certain disorder; and all his subsequent generations will continue having the same disorder in their systems. The only hope and prayer is that when one is exposed to these radiations, the exposure is too small to cause any serious damage.

When an atom bomb explodes nearby, the radiations are extremely intense and sudden. This causes immediate death and destruction of life, pathologically, and damage to the heredity, by genetic damage.

5.25 CATHODE RAY

A cathode-ray tube (CRT) is a vacuum tube in which an electron beam, deflected by applied electric or magnetic fields, produces a trace on a fluorescent screen.

Cathode Ray Tube

The cathode ray tube (CRT), invented in 1897 by the German physicist Karl Ferdinand Braun, is an evacuated glass envelope containing an electron gun a source of electrons and a fluorescent light, usually with internal or external means to accelerate and redirect the electrons. Light is produced when electrons hit a fluorescent tube.

The electron beam is deflected and modulated in a manner that allows an image to appear on the projector. The picture may reflect electrical wave forms (oscilloscope), photographs (television, computer monitor), echoes of radar-detected aircraft, and so on. The single electron beam can be processed to show movable images in natural colours.

Cathode Ray Tube

J. J. Thomson designed a glass tube that was partly evacuated, *i.e.,* all the air had been drained out of the building. He then applied a high electric voltage at either end of the tube between two electrodes. He observed a particle stream (ray) coming out of the negatively charged electrode (cathode) to the positively charged electrode (anode). This ray is called a cathode ray and is called a cathode ray tube for the entire construction.

The experiment Cathode Ray Tube (CRT) conducted by J. J. Thomson, is one of the most well-known physical experiments that led to electron discovery. In addition, the experiment could describe characteristic properties, in essence, its affinity to positive charge, and its charge to mass ratio. This paper describes how J is simulated. J. Thomson experimented with Cathode Ray Tube.

The major contribution of this work is the new approach to modelling this experiment, using the equations of physical laws to describe the electrons' motion with a great deal of accuracy and precision. The user can manipulate and record the movement of the electrons by assigning various values to the experimental parameters.

Apparatus Setup

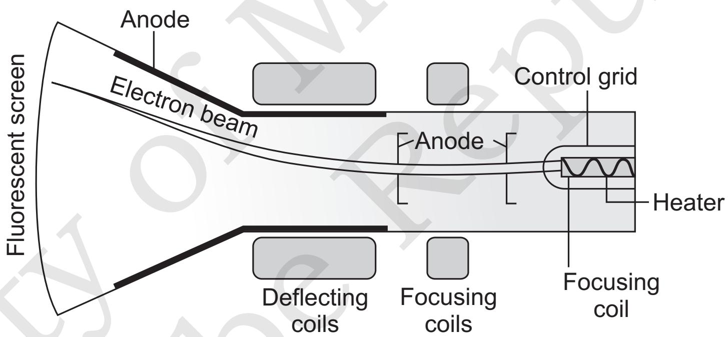


Fig. 5.4. A Diagram of JJ.Thomson Cathode Ray Tube Experiment showing Electron Beam – A cathode-ray tube (CRT) is a large, sealed glass tube.

The apparatus of the experiment incorporated a tube made of glass containing two pieces of metals at the opposite ends which acted as an electrode. The two metal pieces were connected with an external voltage. The pressure of the gas inside the tube was lowered by evacuating the air.

Uses of Cathode Ray Tube

1. Used as a most popular television (TV) display.
2. X-rays are produced when fast-moving cathode rays are stopped suddenly.

3. The screen of a cathode ray oscilloscope, and the monitor of a computer, are coated with fluorescent substances. When the cathode rays fall off the screen are visible on the screen.

5.26 BASIC PHYSICS OF X-RAY

In X-ray diagnostics, radiation that is partly transmitted through and partly absorbed in the irradiated object is utilised. An X-ray image shows the variations in transmission caused by structures in the object of varying thickness, density or atomic composition. In Fig. 5.9, the necessary attributes for X-ray imaging are shown: X-ray source, object (patient) and a radiation detector (image receptor).

After an introductory description of the nature of X-rays, the most important processes in the X-ray source, the object (patient) and radiation detector for the generation of an X-ray image will be described.

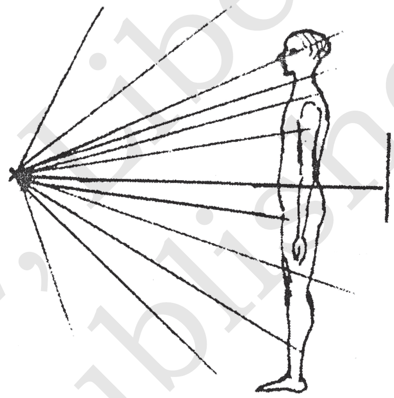


Fig. 5.5. The necessary attributes for X-ray imaging: X-ray source, object (patient) and radiation detector

The Physics of the X-ray Source: The X-ray Tube

(*a*) The nature of X-rays: X-rays are like radio waves and visible light electromagnetic radiation. X-rays, however, have higher frequency, ν, and shorter wavelength, λ, than light and radio waves. The radiation can be considered as emitted in quanta, photons, each quantum having a well-defined energy, $h\nu$, where h is a physical constant, Plancks constant, and ν is the frequency. The energy of X-ray photons are considerably higher than those of light.

A number of the phenomena, which are observed with X-rays are most conveniently described by the wave properties of the radiation while other phenomena can be more easily understood if the X-rays are considered as being composed of particles (photons) with well-defined energies and momentum. The rest mass of a photon is zero. This means that photons can never be found at rest. All photons move at the same velocity, c, in a vacuum, given by $c = 2.998\ 10^8$ m/s.

(*b*) Relationship between wave length and frequency: The wave length multiplied with the frequency (number of wave lengths per unit time) equals the velocity of light

$$\lambda v = c \qquad ...(1)$$

(*c*) The propagation of X-rays: Similarly to visible light, X-rays propagate linearly. The rays from a point source form a divergent beam. The number of photons passing per unit area perpendicular to the direction of motion of the photons is called the fluence, ϕ . The fluence in a vacuum decreases following the inverse square law, given by

$$\phi(r) = \phi(1)\frac{1}{r^2} \qquad ...(2)$$

where r is the distance from the point source and $\phi(1)$ is the fluence at $r = 1$ (relative units). The inverse square law is illustrated in Fig. 5.10.

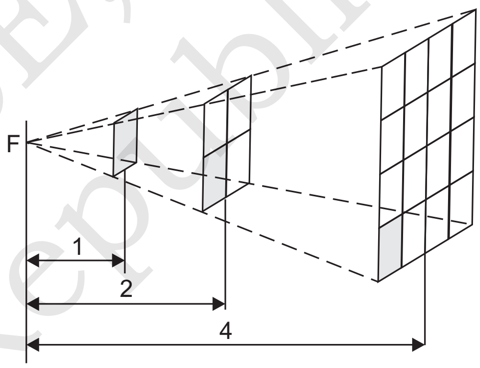


Fig. 5.6. The fluence, ϕ, of X-rays decreases with the square of the distance from the source.

(*d*) Refraction of X-rays: When visible light passes from one medium to another it is refracted due to the different velocities of the rays in different media and interference of waves. The velocity of propagation of X-rays varies much less in different materials and the refraction of X-rays is negligible. For this reason, X-rays cannot be focussed be means of lenses.

(*e*) Diffraction of X-rays: Another wave phenomenon is diffraction. This means that the wave can be bent when passing an edge or a slit. The slit can then be regarded as a new source of waves propagating in all directions. If there is a periodic system of slits (lattice), interference effects will occur. That is, waves which are in phase will be amplified and those that are out of phase will be weakened. In order to demonstrate diffraction with X-rays the lattice constant (distance between the scattering slits) must be of the order of 0.1 nm. Such distances exist between the atomic planes in crystals. Crystals are frequently used for X-ray spectrometry.

***(f)* Generation of X-rays:** An X-ray tube consists of two electrodes, one negative, glow cathode, which upon being heated emits electrons, and one positive, anode. The electrodes are incapsuled in a vacuum. By applying an acceleration potential (20-200 kV), the electrons are accelerated towards the anode. The electrons gain kinetic energy which is the product of their charge and the potential difference. As a measure of the kinetic energy of the electrons and X-ray photons, the unit of 1 eV is used.

Definition: One electron volt (1 eV) is the kinetic energy, that a charged particle of one elementary charge (the charge of an electron) achieves when being accelerated in a potential difference of one volt (1 V); 1 eV = 1.602 10^{-19} J (joule). (See Fig. 5.7).

If the potential difference is 100 kV, each electron gets a kinetic energy of 100 keV (1000 eV = 1 keV).

When the electron reaches the anode it imparts the main part of its energy to the atoms of the anode by ionisations and excitations. This energy will finally appear as heat energy. If an electron passes close to an atomic nucleus, it will change its direction of motion, *i.e.,* exhibits an acceleration. At each such acceleration there is a small probability that the electron looses energy in the form of a photon, Fig. 5.8. These photons are called bremsstrahlung photons and constitute the main part of the X-rays being used in X-ray diagnostic imaging.

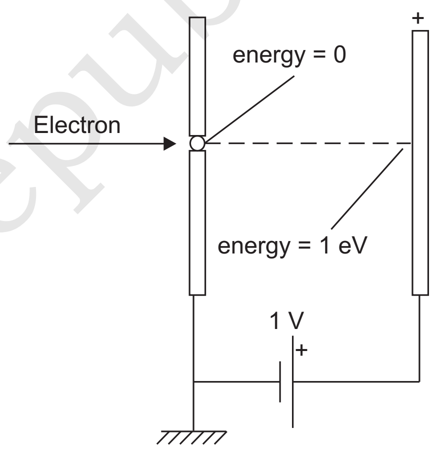


Fig. 5.7. One electron volt (1 eV) is the kinetic energy of an electron which has been accelerated through a potential difference of 1 volt.

The bremsstrahlung photon can obtain an arbitrary energy between zero and the whole of the kinetic energy of the electron, T.

$$h\nu_{max} = T \quad \ldots(3)$$

The relative amount of bremsstrahlung emitted increases with increasing electron kinetic energy and with increasing atomic number, Z, of the anode material.

Since the major part of the energy of the electrons is converted into heat in the anode (about 1% will appear as X-rays), the anode material should have a high melting point and good heat conduction ability. To get a high relative amount of X-ray energy, the anode material should be of high atomic number. Tungsten is the dominating anode material and is in modern X-ray tubes often mixed with renium (Z_W = 74; Z_{Re} = 75).

Modern X-ray imaging requires a small focal spot and high X-ray fluence rates (number of photons per unit area and unit time). To meet these requirements, technical solutions with a line shaped focal spot and rotating anode have been introduced.

The Energy Spectrum of X-rays

(*a*) Dependence of the energy spectrum on tube potential: Fig. 5.13 shows energy spectra from an X-ray tube, the glass envelope of which gives a filtration corresponding to 2 mm Al (aluminium). The X-rays are then additionally filtered by an extra layer of 1 mm Al. The energy spectra show the number of photons per unit energy interval, (keV), emitted within a unit interval of the solid angle, (steradian), when the charge 1 mAs passes through the X-ray tube. The energy spectra have been measured at constant acceleration potential differences of 40, 70, 100 and 130 kV (Mika and Reiss 1969). As can be seen from Fig. 5.9, there are only few photons close to the maximum energy.

The number of X-rays emitted in the anode per unit energy interval increases with decreasing energy.

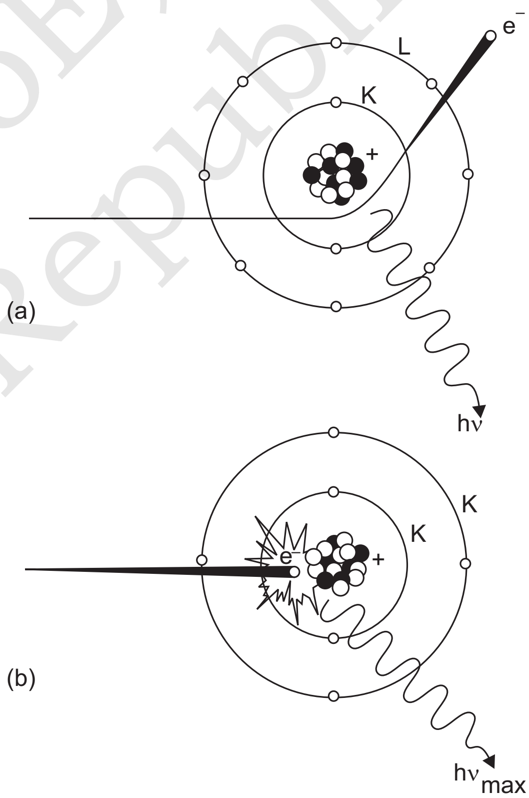


Fig. 5.8. Bremsstrahlung is generated when an electron with high energy changes its direction of motion in the neighbourhood of an atomic nucleus and thereby looses energy.

The attenuation of the photons in the anode itself, the glass envelope and additional filter increases, however, still more with decreasing energy such that the number of low energy photons is heavily reduced. There are practically no photons with energies less than 10 keV, which are emitted from an X-ray tube with the above mentioned filters (Fig. 5.9).

The sharp peaks shown in the energy spectra at 100 and 130 kV acceleration potential differences (Fig. 5.9) are characteristic Kα and Kβ photons from tungsten.

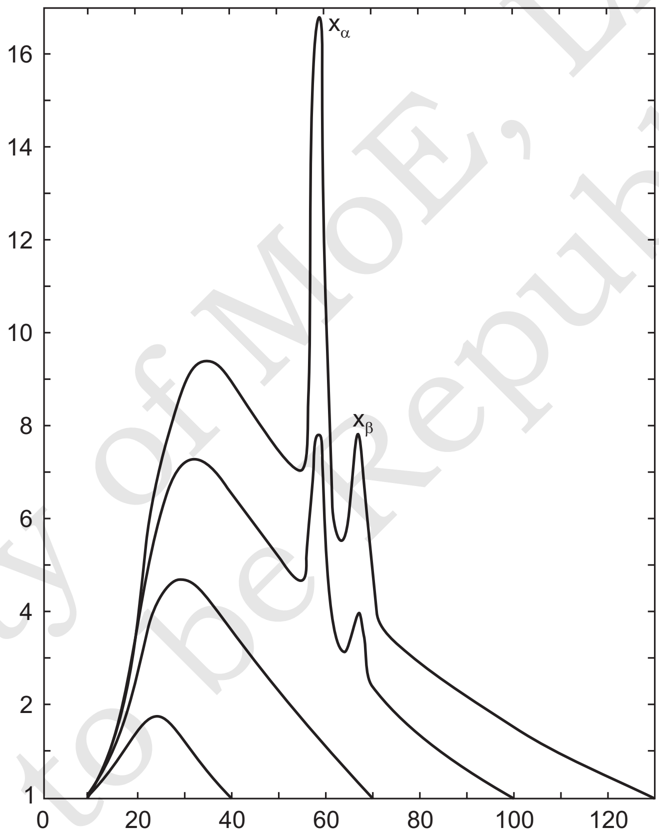


Fig. 5.9. Energy spectra of X-rays at different (constant) acceleration potential differences (Mika and Reiss 1969). Total filtration 3 mm Al

Characteristic roentgen rays (fluorescence radiation) are emitted when a vacancy in an electron shell (here the K-shell) is filled with an electron from an outer shell. The emitted energy equals the difference in binding energy of the electron in the two shells. Vacancies in the K-shell can result from either ionisations caused by the accelerated electrons or from photoelectric absorption of bremsstrahlung photons (with energies higher than the binding energy of the electrons in the K shell) in the anode itself. In order to ionise the K-shell of tungsten an energy of 69.5 keV is needed. For characteristic K-photons to be emitted, the acceleration potential difference must exceed 70 kV.

From Fig. 5.9 it can be seen that the relative proportion of characteristic K-radiation increases with increasing tube potential. This means that the imaging properties of the X-rays is only slowly varying with variations in the tube potential at tube potentials above 130 kV. Fig. 5.9 shows how the number of photons varies at constant value of the tube charge (mAs-value = the product of tube current, mA and exposure time, s).

If instead the acceleration potential difference is kept constant and the charge through the X-ray tube (mAs) is increased, the shape of the energy spectrum remains the same, *i.e.*, the relationship between the number of photons in the different energy intervals. The number of photons in each interval is proportional to the mAs-value.

REVIEW EXERCISE

A. MULTIPLE CHOICE QUESTIONS (MCQs)

1. In the nuclear decay given below :

$${}^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}Y \longrightarrow {}^{A-4}_{Z-1}B^{*} \longrightarrow {}^{A-4}_{Z-1}B,$$

the particles emitted in the sequence are :

(*a*) α, β, γ (*b*) β, α, γ (*c*) γ, β, α (*d*) β, γ, α.

2. An element A decays into an element C by a two step process

A → B + ${}^{4}_{2}$He and B → C + 2e^{-}.

Then,

(*a*) A and C are isotopes. (*b*) A and C are isobars.

(*c*) B and C are isotopes. (*d*) A and B are isobars.

3. If the radius of a nucleus of ^{256}X is 8 fermi, then the radius of ^{4}He nucleus will be

(*a*) 16 fermi (*b*) 2 fermi (*c*) 32 fermi (*d*) 4 fermi.

4. A radioactive element x converts into another stable element y. Half-life of x is 2 h, initially only x is present. After time t, the ratio of atoms of x and y is found to be 1 : 4, then t in hour is

(*a*) 2 (*b*) 4

(*c*) between 4 and 6 (*d*) 6.

5. A radioactive isotope has a half-life of 2 yr. How long will it take the activity to reduce to 3% of its original value?

(*a*) 4.8 yr (*b*) 7 yr (*c*) 10 yr (*d*) 9.6 yr.

6. A radioactive isotope A with a half-life of 1.25×10^{10} years decays into B which is stable. A sample of rock from a planet is found to contain both A and B present in the ratio 1 : 15. The age of the rock is (in years)

(*a*) 9.6×10^{10} (*b*) 4.2×10^{10} (*c*) 5×10^{10} (*d*) 1.95×10^{10}.

7. A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to 1% of its original value?

(*a*) 3.2T years (*b*) 4.6T years (*c*) 6.6T years (*d*) 9.2T years.

8. The density of a nucleus of mass number A is proportional to

(*a*) A^3 (*b*) $A^{1/3}$ (*c*) A^1 (*d*) A^0.

9. The fraction of a sample of radioactive nuclei that remains undecayed in one mean life is

(*a*) $\frac{1}{e}$ (*b*) $1 - \frac{1}{e}$ (*c*) $\frac{1}{e^2}$ (*d*) $1 - \frac{1}{e^2}$.

10. Half-life of a radioactive substance is 20 minute. The time between 20% and 80% decay will be

(*a*) 20 min (*b*) 30 min (*c*) 40 min (*d*) 25 min.

B. FILL IN THE BLANKS

1. The nucleus which has radius one-third of the radius of ^{189}Os is ....

2. If half-life of radio isotope is 2 second and number of atoms is only 4, then after one half-life, the remaining atoms are ....

3. The energy released by the fission of one uranium atom is 200 MeV. The number of fissions per second required to produce 3.2 W of power is .... (Take 1 eV = 1.6×10^{-19} J)

4. The half-life of a radioactive isotope X is 50 years. It decays to another element Y which is stable. The two elements X and Y were found to be in the ratio of 1 : 15 in a sample of a given rock. The age of the rock was estimated to be ....

5. The power obtained in a reactor using ^{235}U disintegration is 1000 kW. The mass decay of ^{235}U per hour is ....
6. A radioactive nucleus of mass M emits a photon of frequency n and the nucleus recoils. The recoil energy will be ....
7. If the binding energy per nucleon of deuteron is 1.115 MeV, its mass defect in atomic mass unit is ....
8. A uranium nucleus $^{238}_{92}U$ emits an α-particle and β-particle in succession. The atomic number and mass number of the final nucleus will be ....
9. In the nuclear reaction $^{14}_{7}N + X \longrightarrow\ ^{14}_{6}C +\ ^{1}_{1}H$, the X will be ....
10. If the nuclear radius of ^{27}Al is 3.6 fermi, the approximate nuclear radius of ^{64}Cu in fermi is ....

C. VERY SHORT ANSWER TYPE QUESTIONS

1. Name three nuclei which are on the 'bottom points' of binding energy curve.
2. Name five nuclei which lie on the peaks in binding energy curve.
3. Why electron capture is more common in heavy atoms?
4. How many joule are contained in 1 kWh?
5. What exactly makes large nuclei unstable?
6. What is one roentgen?
7. Name two elementary particles which have almost infinite life time.
8. Is free neutron a stable particle?
9. Cadmium rods are provided in a reactor. Why?
10. Name one physical quantity which can be said to be the source of binding energy.

D. SHORT ANSWER TYPE QUESTIONS

1. In a natural uranium reactor, heavy water is a preferred moderator to ordinary water.
2. Very high temperatures as those obtained in the interior of the sun are required for fusion reaction to take place.
3. The half-lives of radioactive nuclides that emit α-rays vary from microsecond to billion year. What is the reason for this large variation in the half-life of α-emitters ?
4. Which of the two is more stable—$^{7}_{3}Li$ or $^{4}_{3}Li$?
5. It is said that a very powerful crane is required to lift a nuclear mass of microscopic size. Comment on this.

E. LONG ANSWER TYPE QUESTIONS

1. Explain, with the help of a nuclear reaction in each of the following cases, how the neutron to proton ratio changes during (*i*) alpha decay (*ii*) beta decay?

2. The nucleus of an atom of ${}^{235}_{92}Y$, initially at rest, decays by emitting an α-particle as per the equation :

$${}^{235}_{92}Y \longrightarrow {}^{231}_{90}x + {}^{4}_{2}He + \text{Energy}$$

3. Prove that the instantaneous rate of change of the activity of a radioactive substance is inversely proportional to the square of its half-life.

4. A radioactive material is reduced to $\frac{1}{16}$th of its original amount in 4 days. How much material should one begin with so that 4×10^{-3} kg of the material is left after 6 days.

5. Define the term 'Activity' of a radioactive substance. State its SI unit. Two different radioactive elements with half-lives T_1 and T_2 have N_1 and N_2 (undecayed) atoms respectively present at a given instant. Determine the ratio of their activities at this instant.

PERIOD-6

P12CH6

TOPIC

6

High Energy Physics

6.1 PARTICLE PHYSICS

Particle physics or **high energy physics** is the study of fundamental particles and forces that constitute matter and radiation. The fundamental particles in the universe are classified in the Standard Model as fermions (matter particles) and bosons (force-carrying particles). There are three generations of fermions, but ordinary matter is made only from the first fermion generation. The first generation consists of up and down quarks which form protons and neutrons, and electrons and electron neutrinos. The three fundamental interactions known to be mediated by bosons are electromagnetism, the weak interaction, and the strong interaction.

Quarks cannot exist on their own but form hadrons. Hadrons that contain an odd number of quarks are called baryons and those that contain an even number are called mesons. Two baryons, the proton and the neutron, make up most of the mass of ordinary matter. Mesons are unstable and the longest-lived last for only a few hundredths of a microsecond. They occur after collisions between particles made of quarks, such as fast-moving protons and neutrons in cosmic rays. Mesons are also produced in cyclotrons or other particle accelerators.

Particles have corresponding antiparticles with the same mass but with opposite electric charges. For example, the antiparticle of the electron is the positron (also known as an anti-electron). The electron has a negative electric charge, the positron has a positive charge. These antiparticles can theoretically form a corresponding form of matter call antimatter. Some particles, such as the photon, are their own antiparticle.

These elementary particles are excitations of the quantum fields that also govern their interactions. The dominant theory explaining these fundamental particles and fields, along with their dynamics, is called the Standard Model. The reconci1iation of gravity to the current particle physics theory is not solved; many theories have addressed this problem, such as loop quantum gravity, string theory and supersymmetry theory.

Practical particle physics is the study of these particles in radioactive processes and in particle accelerators such as the Large Hadron Collider. Theoretical particle physics is the study of these particles in the context of cosmology and quantum theory. The two are closely interrelated: the Higgs boson was postulated by theoretical particle physicists and its presence confirmed by practical experiments.

6.2 STANDARD MODEL

The current state of the classification of all elementary particles is explained by the Standard Model, which gained widespread acceptance in the mid-1970s after experimental confirmation of the existence of quarks. It describes the strong, weak, and electromagnetic fundamental interactions, using mediating gauge bosons. The species of gauge bosons are eight gluons, W^-, W^+ and Z bosons and the photon. The Standard Model also contains 24 fundamental fermions (12 particles and their associated anti-particles), which are the constituents of all matter. Finally, the Standard Model also predicted the existence of a type of boson known as the Higgs boson. On 4 July 2012, physicists with the Large Hadron Collider at CERN announced they had found a new particle that behaves similarly to what is expected from the Higgs boson.

The Standard Model, as currently formulated, has 61 elementary particles. Those elementary particles can combine to form composite particles, accounting for the hundreds of other species of particles that have been discovered till date. The Standard Model has been found to agree with almost all the experimental tests conducted to date. However, most particle physicists believe that it is an incomplete description of nature and that a more fundamental theory awaits discovery. In recent years, measurements of neutrino mass have provided the first experimental deviations from the Standard Model, since neutrinos do not have mass in the Standard Model.

6.3 SUBATOMIC PARTICLES

Modern particle physics research is focused on subatomic particles, including atomic constituents, such as electrons, protons, and neutrons (protons and neutrons are composite particles called baryons, made of quarks), that are produced by radioactive and scattering processes; such particles are photons, neutrinos, and muons as well as a wide range of exotic particles. All particles and their interactions observed to date can be described almost entirely by the Standard Model.

Elementary Particles

<table>
<tr><th></th><th>Types</th><th>Generations</th><th>Antiparticle</th><th>Colours</th><th>Total</th></tr>
<tr><td>Quarks</td><td rowspan="2">2</td><td rowspan="2">3</td><td>Pair</td><td>3</td><td>36</td></tr>
<tr><td>Leptons</td><td>Pair</td><td>None</td><td>12</td></tr>
<tr><td>Gluons</td><td rowspan="5">1</td><td rowspan="5">None</td><td>Own</td><td>8</td><td>8</td></tr>
<tr><td>Photon</td><td>Own</td><td rowspan="4">None</td><td>1</td></tr>
<tr><td>Z Boson</td><td>Own</td><td>1</td></tr>
<tr><td>W Boson</td><td>Pair</td><td>2</td></tr>
<tr><td>Higgs</td><td>Own</td><td>1</td></tr>
<tr><td colspan="5">Total number of (known) elementary particles:</td><td>61</td></tr>
</table>

Dynamics of particles are also governed by quantum mechanics; they exhibit wave-particle duality, displaying particle-like behaviour under certain experimental conditions and wave-like behaviour in others. In more technical terms, they are described by quantum state vectors in a Hilbert space, which is also treated in quantum field theory. Following the convention of particle physicists, the term *elementary particles* is applied to those particles that are, according to current understanding, presumed to be indivisible and not composed of other particles.

Bosons

Bosons are the mediators or carriers of fundamental interactions, such as electromagnetism, the weak interaction, and the strong interaction. Electromagnetism is mediated by the photon, the quanta of light. The weak interaction is mediated by the W and Z bosons. The strong interaction is mediated by the gluon, which can link quarks together to form composite particles. Due to the aforementioned color confinement, gluons are never observed independently. The Higgs boson gives mass to the W and Z bosons via the Higgs mechanism – the gluon and photon are expected to be massless. All bosons have an integer quantum spin (0 and 1) and can have the same quantum state.

Antiparticles and Color Charge

Most aforementioned particles have corresponding antiparticles, which compose antimatter. Normal particles have positive lepton or baryon number, and antiparticles have these numbers negative. Most properties of corresponding antiparticles and particles are the same, with a few gets reversed; the electron's antiparticle, positron, has an opposite charge. To differentiate between antiparticles and particles, a plus or negative sign is added in superscript. For example, the electron and the positron are denoted e^- and e^+. When a particle and an antiparticle interacts with each other, they are annihilated and convert to other particles. Some particles have no antiparticles, such as the photon or gluon.

Quarks and gluons additionally have color charges, which influences the strong interaction. Quark's color charges are called red, green and blue (though the particle itself have no physical color), and in antiquarks are called antired, antigreen and antiblue. The gluon can have eight color changes, which are the result of quarks' interactions to form composite particles (gauge symmetry SU(3)).

6.4 INADEQUACY OF CLASSICAL MECHANICS

The development of classical mechanics is based on Newton's three laws (*i*) the law of inertia (*ii*) law of force (*iii*) the law of action and reaction. These laws include the concept of absolute mass, absolute space and absolute time. The classical mechanics explains correctly the motion of celestial bodies like planets, stars and macroscopic as well as microscopic terrestrial bodies moving with non-relativistic speeds (*i.e.,* $v << c$, c being the speed of light in vacuum).

(*i*) The necessity for a departure from classical mechanics is clearly shown by experimental results.

The forces known in classical electrodynamics are inadequate for the explanation of the remarkable stability of atoms and molecules, which is necessary in order that materials may have definite physical and chemical properties at all.

(*ii*) It does not hold in the region of atomic dimensions *i.e.,* it cannot explain the non-relativistic motion of atoms, electrons, protons etc.

(*iii*) It could not explain observed spectrum of black body radiations, the observed variation of specific heat of metals and gases.

(*iv*) It could not explain the origin of discrete spectra of atoms since according to classical mechanics the energy changes are always continuous. This difficulty was later on resolved by Bohr to some extent.

In spite of this classical mechanics could not explain a large number of observed phenomenon like photoelectric effect, Compton effect, Raman effect, etc.

6.5 BLACK BODY RADIATION

A perfectly black body is one which absorbs totally all the radiation of any wavelength which fall on it. Such a body does not reflect any radiation and so it appears black. Experimentally, such a body is represented by a hollow container with a small hole in the wall. When such a body is heated, it emits radiations of all possible wavelengths. These radiations are independent of the nature of the substance. Such heat radiations in a uniform temperature enclosure are known as **black body radiations**. Lummer and Pringsheim (1899) made experiments to determine the distribution of energy among radiations of different wavelengths emitted by a black body at various temperatures. Fig. 6.1 shows the variation of intensity of radiation with the wavelengths at different temperatures. A close investigation reveals the following important facts.

(*i*) At a given temperature, the energy is not uniformly distributed in the radiation spectrum.

(*ii*) At a given temperature, the intensity of radiation increases with increase in wavelength and becomes maximum at a particular wavelength. With further increase in wavelengths the intensity of radiation decreases.

(*iii*) An increase in temperature causes a decrease in λ_m the wavelength for which energy emitted is maximum.

(*iv*) For all wavelengths, an increase in temperature causes an increase in the energy emission.

(*v*) The area under each curve represents the total energy emitted for the complete spectrum at a particular temperature.

(*a*) In order to explain the observed spectra by applying the classical thermodynamics, it was shown by Wein that the amount of energy contained in the spectral region included with in the wavelength λ and $\lambda + d\lambda$ emitted by a black body is given by,

$$E_\lambda d\lambda = \frac{A}{\lambda^5} e^{-\frac{C}{\lambda T}} . d\lambda \quad \ldots(1)$$

where A and C are constants.

This formula works well only for short wavelengths, but did not reproduce the results at long wavelengths and high temperatures. Equation (1) gives finite energy even for T = ∞. Lord Rayleigh argued that it is, unlikely that E should be finite for infinite value of temperature.

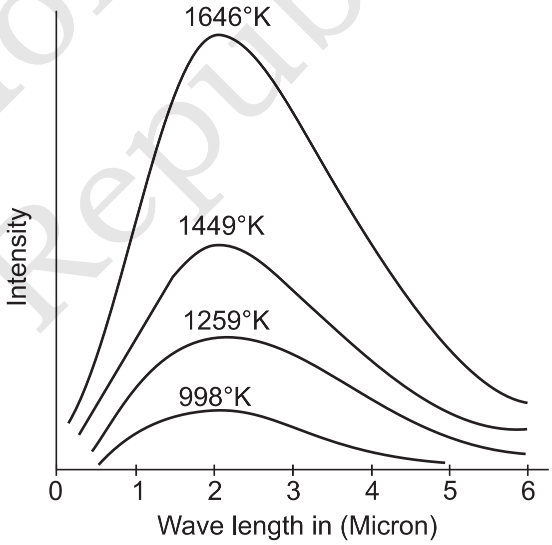


Fig. 6.1

(*b*) **Rayleigh and Jeans,** by assuming that the radiation in black body have degrees of freedom and applying the law of equipartition of energy showed that

$$E_\lambda d\lambda = \frac{A}{\lambda^4} T d\lambda \quad \ldots(2)$$

where A is a constant.

It is clear from this formula that the energy radiated in a given wavelength range $d\lambda$ increases rapidly as λ decreases and approaches infinity for very short wavelengths, which cannot be true. Thus the formula (2) holds good in the region of longer wavelengths but fails for the shorter wavelengths. Thus, Wien's law and Rayleigh Jean's law do not precisely agree with the experimental results.

Planck's Radiation Law

In order to explain the spectrum of black body radiation, Planck put forward his quantum hypothesis, according to which a black body contains simple harmonic oscillators which are capable of vibrating with all possible frequencies. The frequency of a radiation emitted by an oscillator is the same as the frequency of its vibration. An oscillator cannot emit energy in a continuous manner, it can emit energy in the multiples of the unit called quantum. If an oscillator is vibrating with a frequency ν, it can only radiate in quanta of magnitude $h\nu$, where h is a constant, called Planck's constant and its value is 6.625×10^{-27} erg-sec.

If N is the total number of Planck's oscillator and E is their total energy, then the average energy per oscillator is given by,

$$\bar{\in} = \frac{E}{N} \qquad \ldots(3)$$

Let N_0, N_1, N_2, ..., N_n etc. be the number of oscillators having energies 0, $\in = h\nu$, $2\in$, ..., $n\in$ etc. respectively, then

$$N = N_0 + N_1 + \ldots + N_n = \sum_{n=0}^{\infty} N_n \qquad \ldots(4)$$

and

$$E = \in[N_1 + 2N_2 + \ldots + nN_n + \ldots) = \sum_{n=0}^{\infty} n\in N_n \qquad \ldots(5)$$

According to Maxwell's distribution formula, the probability for an oscillator to possess an energy E is given by,

$$\text{Exp}\,[-E/kT]$$

Hence the average energy per oscillator can be written as

$$\bar{\in} = \frac{E}{N} = \frac{\sum_{n=0}^{\infty} n\in e^{-\frac{n\in}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{n\in}{kT}}} \qquad \ldots(6)$$

Simplifying we get,

$$\bar{\in} = \frac{h\nu}{e^{h\nu}kT - 1} \qquad \ldots(7)$$

Now, the number of oscillators per unit volume in the frequency range ν and ν + *d*ν is given by,

$$N = \frac{8\pi\nu^2 d\nu}{e^3} \qquad \ldots(8)$$

Multiplying it by the average energy per oscillator, given by eqn. (7), we get the total energy per unit volume belonging to the range *d*ν or the energy density belonging to range *d*ν as

$$E_\nu d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}d\nu \qquad \ldots(9)$$

This is known as Planck's radiation law.

Planck's radiation law explains all the observed facts of the black body spectrum for the entire wavelength range.

Above description illustrates that Planck's law is perfect one and all other laws follow as special cases of this law, which account only for limited portion of the black body spectrum.

6.6 QUANTUM THEORY OF RADIATION AND PHOTON

The quantum theory of radiation was proposed by Max Planck to explain the distribution of energy with wavelength in black body radiation. He put forward the hypothesis of atomicity of energy and introduced his **quantum** of action. This theory, has been able to explain a number of phenomenona concerning the interaction of energy with matter. According to this theory, an accelerated electron, which Planck described as a **linear oscillator** does not radiate energy continuously as required by the electromagnetic theory of light but the energy is emitted in tiny packets or **quanta.**

The oscillator emit energy only when it passes from a higher energy state to lower energy state and absorbs energy when it goes from a lower energy state to a higher energy state. No emission or absorption of energy takes place when the oscillator is in a given state. The smallest amount of energy which can be emitted or absorbed by the oscillator is $h\nu$. In other words a radiation of frequency ν is emitted as quantum of energy $E = h\nu$ where h is Planck's universal constant of action having a value equal to 6.62×10^{-34} J sec. The quantum is the basic unit of energy and cannot be subdivided. It is the atom of energy and is known as **Photon.**

This theory has been able to explain a number of phenomena which could not be explained according to the classical concept of radiation. For example, it gives a very satisfactory explanation of variation of specific heat of solids with temperature, photo electric effect, Compton effect etc. and has been successfully applied by Bohr in the theory of hydrogen spectrum.

(*a*) **Energy of photon.** Energy of photon is only in multiples of $h\nu$, where h is Planck's constant and ν its frequency. If a photon undergoes interaction with matter, either it can be completely absorbed, transferring all its energy or it may transfer part of its energy, and its frequency is adjusted to a lower value thereby, maintaining its particle character. If many photons exist, they have more energy and intensity of radiation is also large. It means that intensity is not concerned with the individual photon energy but simply gives their number. Hence, energy is only dependent on the intrinsic property of the photons. Thus the energy of photon is independent of its intensity, depending only on its frequency, a concept contrary to classical ideas where radiation is considered purely as waves and energy estimated by the intensity of the wave disturbance, dependent on the physical properties of the medium.

(*b*) **Mass and momentum of photon.** As photons have energy ($E = h\nu$) and are in motion with velocity c. According to the theory of relativity a mass m has an energy equivalent to mc^2.

Hence $$E = mc^2$$

$$m = E/c^2$$

$$\text{Mass of photon} = h\nu/c^2 \qquad \ldots(10)$$

$$\text{As momentum} = \text{mass} \times \text{velocity}$$

$$= \frac{h\nu}{c^2} \times c = \frac{h\nu}{c} = \frac{h}{\lambda} \qquad \ldots(11)$$

where λ is the wavelength of the radiation.

A photon is a partide of zero rest mass. Its dynamical variables are energy ($h\nu$) and momentum (h/λ). A photon has zero charge and spin equal to one quantum unit $\left(\frac{h}{2\pi} = h\right)$. Two charges or two magnetic poles exert a force on each other due to exchange of photons. A photon interacts with all the charged particles and can also interact with some neutral particles.

A photon behaves as a small elastic sphere for all purposes. When a photon collides with an electron both momentum and energy are conserved as in the case of collision between two electric spheres. This fact has been verified by Compton in his experiments on X-rays. When X-rays of frequency ν collide with an electron of mass *m*, then

$$h\nu = \frac{1}{2}mv^2 + h\nu' \qquad \text{...(12)}$$

where v is the velocity with which electron moves and ν′ is the frequency of the new radiation given out.

(*c*) Non-electrical nature of photons. The photons constituting radiations are electrically neutral. They are not affected by electric or magnetic fields and also do not ionise directly by themselves. However, they can eject charged particles from matter, when they impinge on atoms.

6.7 PHOTO-ELECTRIC EFFECT

Liberation of electrons from matter under the influence of sufficiently high frequency electromagnetic radiations is known as photo-electric effect and emitted electrons, are called photo-electrons.

Experimental Arrangement

The experimental arrangement to demonstrate photo-electric effect is shown in Fig. 6.2

It consists of an evacuated glass tube T fitted with a photo sensitive plate A and a quartz window W for allowing the ultra violet light to fall on the plate A from a source S. Another plate B is also sealed at the other end of the tube. The middle point of the H.T battery and the plate A are earth connected, so that the potential of plate A is zero. The plate B can be given a positive or a negative potential by the potential dividing arrangement as shown in Fig. 6.2.

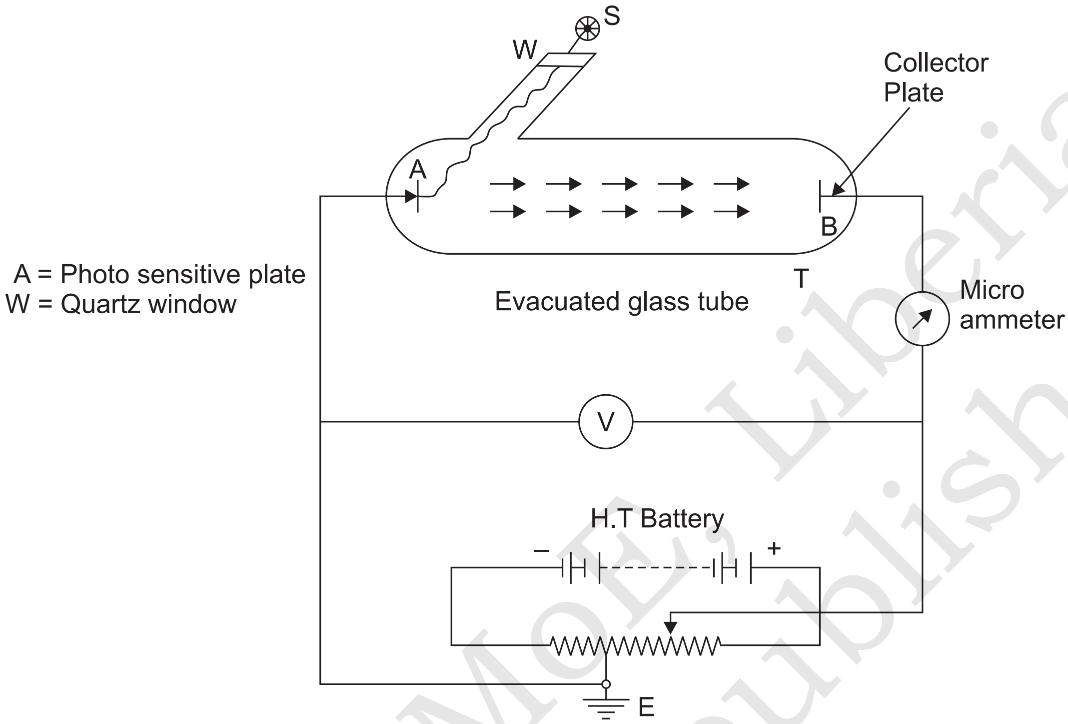


Fig. 6.2

When light of suitable frequency is allowed to fall on the plate A from a source S through the quartz window W and the plate B is kept at a positive potential an electric current flows in the outer circuit as indicated by the micro-ammeter. This is due to photo electrons emitted from the plate A which are attracted towards the plate B as shown. Some important results are as under.

(*a*) Intensity effect. For a given metal rate of emission of photo-electrons *i.e.*, photo current is directly proportional to the intensity of incident radiation for a given light Fig. 6.3 provided the frequency is above the threshold frequency *i.e.,* a bright light always gives more photo current than a dim one for a given frequency.

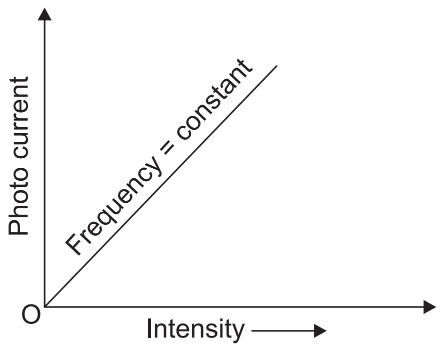


Fig. 6.3

(*b*) Frequency effect. For a given metal maximum kinetic energy of photoelectrons varies linearly with the frequency of incident radiation provided it is greater than the threshold frequency and is independent of its intensity *i.e.,* blue light will always give more energetic photo electrons than red, what ever be its intensity.

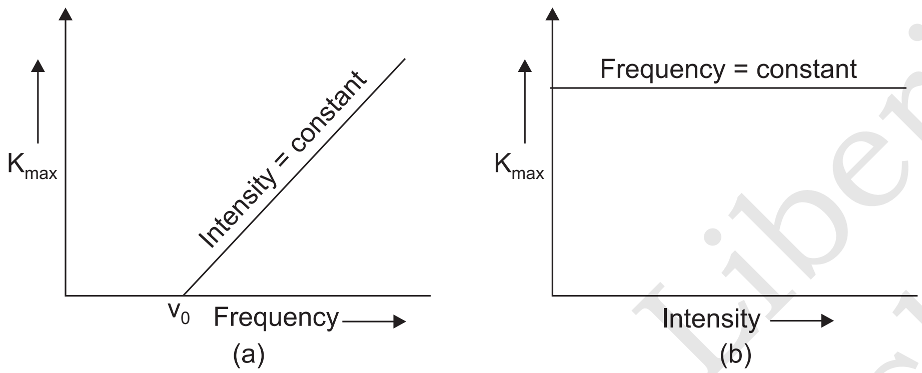


Fig. 6.4

(*c*) Effect of nature of metal. If light of different frequencies in turn is incident on a given metal, photoelectric effect takes place only if the frequency of incident radiation is more (or wavelength is less) than a specific value ν_0. This specific value of frequency (ν_0) is called threshold frequency or cutoff frequency and depends only on the nature of metal.

(*d*) Time-delay effect. Within the limits of experimental accuracy (about 10^{-9} sec), there is no time log between incidence of radiation and emission of photoelectrons *i.e.,* as soon as light is incident on the metal, photo electrons are emitted.

6.8 FAILURE OF (CLASSICAL) ELECTROMAGNETIC THEORY

According to the electromagnetic theory of light, a substance exposed to light is subjected to an oscillatory electric field, the intensity of light being proportional to the square of the amplitude of electric vector. The electrons within the substance interact with the field, gain kinetic energy, and leave the substance as soon as the kinetic energy exceeds the binding energy. The electromagnetic energy gained by the electron depends upon

(*i*) intensity of incident light.

(*ii*) area of the surface over which light falls

(*iii*) effective area of the electron exposed to light and

(*iv*) time for which the surface is illuminated.

If we consider the light falling on a sodium surface, a detectable photo-electric current will be obtained when 10^{-6} W/m^2 of electromagnetic energy is absorbed by the surface. A layer of sodium one atom thick and 1 m^2 in area has about 10^{19} atoms. If we suppose that the incident light is absorbed by the uppermost layer of sodium atoms, each atom will receive an average amount of energy equal to 10^{-25} W so that 1.6×10^6 sec will be required by an atom to have 1 eV of energy and an electron in sodium will gain sufficient energy to be liberated in about 2 months.

Hence the electromagnetic theory fails to explain there is no time lag between the instant the light falls and that of the emission of photo electrons.

Secondly, according to classical electromagnetic theory, the ejection of an electron should depend upon the incident energy *i.e.* , intensity of light and not on frequency. In other words there should be no threshold frequency for any material. This is contrary to the observed facts.

Again, there is no limit on the maximum energy to be transferred to the electron *i.e.*, the electron can have any value of maximum energy. This is contrary to observed facts. Hence classical electromagnetic theory of light fails to explain the basic facts of photo electric effect.

6.9 EINSTEIN'S PHOTO-ELECTRIC EQUATION

In 1905 Einstein proposed a theory based upon Planck's idea of quanta of energy which gave a satisfactory explanation of the various experimental facts. According to him, monochromatic light of frequency ν consists of photons of energy $h\nu$. When a photon with a sufficient energy content strikes an electron of a photo sensitive material, a part of its energy known as the work function W_0 of the surface, is used up in liberating the electron from the surface, whereas, the remaining is spent in imparting kinetic energy to it. If m is the mass and v is the maximum velocity of the emitted electron, then

$$h\nu = W_0 + \frac{1}{2}mv_{max}^2 \qquad \ldots(13)$$

This is known as Einstein's photo-electric equation. The maximum kinetic energy of the emitted electron is given from eq. (13) is

$$\frac{1}{2}mv^2 = (h\nu - W_0) \quad ...(14)$$

It follows from eq. (14) that the maximum velocity of the emitted electron depends upon the frequency of the incident radiation. An increase in the frequency of the incident light increases the amount of energy carried by each individual light quantum so that, during each collision with a free electron in the metal, these quanta impart larger amount of kinetic energy. An increase in the intensity of light cannot cause any change in the kinetic energy (or the maximum velocity) of the emitted electrons. More intense light simply more number of light quanta having the same energy falling on the surface per sec. Since one light quantum can emit only one electron, the number of electrons emitted will correspondingly increase.

Further, if ν_0 is the lowest or threshold frequency which just causes the emission of electrons, then we have

$$h\nu_o = W_0$$

therefore eq. (14) reduces to

$$\boxed{\frac{1}{2}mv^2 = h(\nu - \nu_0)} \quad ...(15)$$

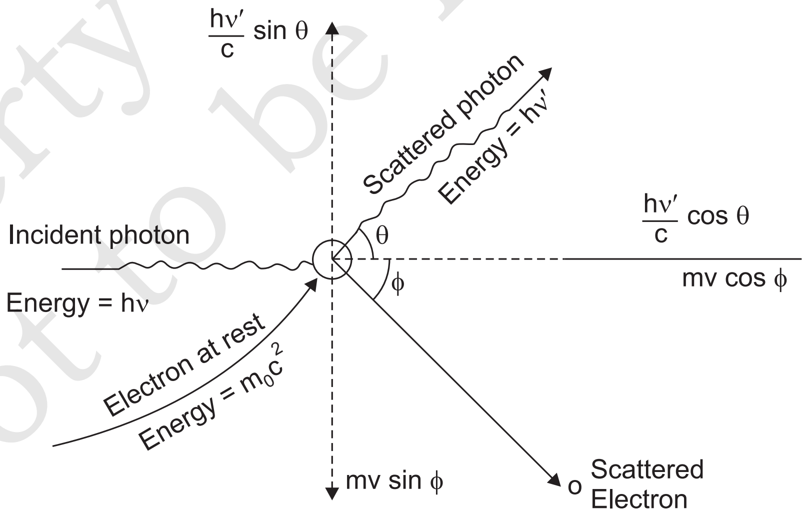


Fig. 6.6

Let us consider the case before the collision

(*i*) Energy of incident photon = $h\nu$

(*ii*) Momentum of incident photon = $\dfrac{h\nu}{c}$

(*iii*) Rest mass of electron = m_0, so that rest mass energy = m_0c^2

(*iv*) Rest mass momentum of electron = 0.

And after the collision

(*i*) Energy of scattered photon = $h\nu'$

(*ii*) Momentum of scattered photon = $\dfrac{h\nu'}{c}$ ...(16)

(*iii*) Mass of the electron moving with velocity $v = m$, so the relativistic energy of electron = mc^2

(*iv*) Momentum of the recoil electron = mv ...(17)

Now applying the principle of conservation of energy, before and after collision, we have

$$h\nu + m_0c^2 = h\nu' + mc^2 \quad \text{...(18)}$$

Again applying the principle of conservation of momentum along the direction of incident photon, before and after the collision, we have

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \quad \text{...(19)}$$

and conservation of momentum ν a direction ⊥ to the direction of incident photon, we have

$$0 + 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \quad \text{...(20)}$$

Now from eq. (19), we have

$$h\nu = h\nu'\cos\theta + mvc\cos\phi$$

or
$$mvc\cos\phi = h\nu - h\nu'\cos\theta \quad \text{...(21)}$$

and From eq. (20), we have

$$mvc\sin\phi = h\nu'\sin\theta \quad \text{...(22)}$$

Squaring (21) and (22) and adding, we get

$$m^2v^2c^2 = (h\nu - h\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2$$

or
$$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\nu'\cos\theta + h^2\nu'^2\sin^2\theta$$
$$= h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta$$
$$= h^2(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta) \quad \text{...(23)}$$

Also from eq. (18), we have

$$mc^2 = h(\nu - \nu') + m_0c^2$$

squaring $$m^2c^4 = h^2(\nu^2 + \nu'^2 - 2\nu'\nu) + m_0^2c^4 + 2hm_0c^2(\nu - \nu') \quad ...(24)$$

Subtracting (23) from (24), we have

or $$m^2c^2\,(c^2 - v^2) = 2h^2\nu'\nu\,(\cos\theta - 1) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$
$$= -\,2h^2\nu'\nu\,(1 - \cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

or $$\frac{m_0^2}{(1 - v^2/c^2)} \times c^2(c^2 - v^2) = 2h^2\nu'\nu\,(1 - \cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

$$\left[\because \quad m = \frac{\mu_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}\right]$$

or $$m_0^2c^4 = -\,2h^2\nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

or $$2h^2\nu\nu'(1 - \cos\theta) = 2h(\nu - \nu')m_0c^2$$

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

or $$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - \cos\theta) \quad ...(25)$$

which may also be written as

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1 - \cos\theta)$$

or $$\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta)$$

$$= \frac{h}{m_0c} \cdot 2\sin^2\theta/2 \quad ...(26)$$

From eq. (26) it is clear that

(*i*) Change in wavelength is independent of the wavelength of the incident photon as well as the nature of the scattering substance but depends only on the angle of scattered photon.

(*ii*) When $\theta = 0$, $\lambda' - \lambda = 0$, *i.e.*, no scattering takes place along the direction of incident photon.

(*iii*) When $\theta = \pi/2$,

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0c} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} = 0.024\ \text{Å}$$

This change in wavelength is called Compton's wavelength is denoted λ_c and as is clear from above it is constant.

(*iv*) When $\theta = \pi$,

$$\lambda - \lambda' = \Delta\lambda = \frac{2h}{m_0 c} = 0.48 \text{ Å}$$

So when θ varies between 0° and π, the wavelength of scattered photon varies between λ to $\lambda + \dfrac{2h}{m_0 c}$, provided the incident photon is of very small wavelength.

Direction of the recoil electron. Dividing eq. (22) by eq. (21), we have

$$\tan\phi = \frac{h\nu' \sin\theta}{h\nu - h\nu' \cos\theta} = \frac{\nu' \sin\theta}{\nu - \nu' \cos\theta} \qquad \text{...(27)}$$

Also from eq. (25), we have

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2}(1 - \cos\theta)$$

or $$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2} \cdot 2\sin^2\frac{\theta}{2}$$

or $$\frac{\nu}{\nu'} = 1 + \frac{h\nu}{m_0 c^2} \cdot 2\sin^2\frac{\theta}{2}$$

or $$\nu' = \frac{\nu}{1 + \left(\dfrac{h\nu}{m_0 c^2}\right) \times 2\sin^2\dfrac{\theta}{2}}$$

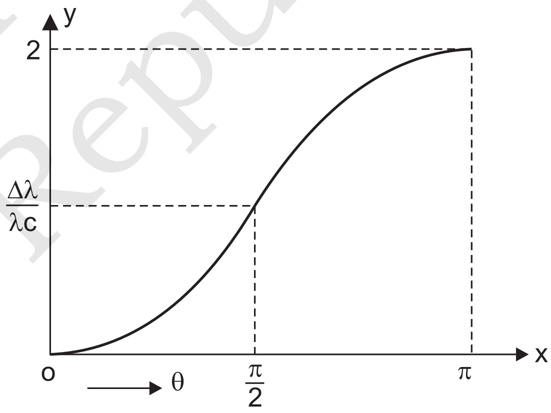


Fig. 6.7

Putting this value of ν' in equation (27), we get

$$\tan\phi = \frac{\dfrac{\nu\sin\theta}{\left(1 + \alpha \cdot 2\sin^2\dfrac{\theta}{2}\right)}}{\nu - \left[\dfrac{\nu}{\left(1 + \alpha \cdot 2\sin^2\dfrac{\theta}{2}\right)}\right]\cos\theta}$$

where $\alpha = \dfrac{h\nu}{m_0 c^2}$

$$\therefore \qquad \tan\phi = \frac{\sin\theta}{1 + 2\alpha\sin^2\frac{\theta}{2} - \cos\theta} = \frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{1 + 2\alpha\sin^2\frac{\theta}{2} - \cos\theta}$$

$$= \frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{(1-\cos\theta) + 2\alpha\sin^2\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} + 2\alpha\sin^2\frac{\theta}{2}}$$

$$\frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}(1+\alpha)} = \frac{\cot\frac{\theta}{2}}{1+\alpha}$$

or
$$\tan\phi = \frac{\cot\frac{\theta}{2}}{\left(1 + \frac{h\nu}{m_0c^2}\right)} \qquad \ldots(28)$$

which gives the direction of the recoil electron in terms of the frequency of the incident photon and the direction of the scattered photon.

Behaviour of photon and electron when θ varies from 0 to π

(*i*) When $\theta = 0,\ \cot\frac{\theta}{2} = \infty$

$$\therefore \qquad \tan\phi = \infty \quad \text{or} \quad \phi = \frac{\theta}{2}$$

Also $\qquad \Delta\lambda = 0$

This means that the photon goes unscattered, while the electron moves at right angles to the direction of the photon.

(*ii*) When $\theta = \frac{\pi}{2}; \cot\frac{\theta}{2} = 1; \tan\phi = \frac{1}{1+\alpha}$

This being positive ϕ lies between 0 and 90°

$$\Delta\lambda = \frac{h}{m_0c} = 0.0242 \text{ Å}$$

This means that photon gets scattered and moves at right angles to the direction of incidence. The change in wavelength is 0.0242 Å while the electron moves in any direction making an angle ϕ less than $\frac{\pi}{2}$ with the direction of incidence.

(*iii*) When $\theta = \pi$; $\cot\frac{\theta}{2} = 0$, $\therefore$ $\tan\phi = 0$, or $\phi = 0$

and $$\Delta\lambda = \frac{2h}{m_0 c} = 0.0484 \text{ Å}$$

This means that the photon reverses its direction with change in wavelength of 0.0484 Å while the electron moves in the direction of the incident photon. Thus, maximum change in wavelength take place when the photon gets scattered in the direction of incidence.

(*iv*) **Electron cannot be scattered at greater than 90° ·**

The angle of scattering for the electron ϕ is related to the angle of scattering for the photon θ as under.

$$\tan\phi = \frac{\cot\frac{\theta}{2}}{1 + \frac{h\nu}{m_0 c^2}}$$

Now $$1 + \frac{h\nu}{m_0 c^2} = a \text{ constant} = k$$

$$\tan\phi = k\cot\frac{\theta}{2}$$

The maximum value of $\cot\frac{\theta}{2} = \infty$.

Hence the maximum value of $\tan\phi = \infty$ or $\phi = \frac{\pi}{2}$.

In other words the electron cannot be scattered at angle greater than 90°.

Kinetic Energy of Recoil Electron

In the compton effect the K.E. of the recoil electron will be equal to the decrease in the energy of the incident photon.

Let the energy of incident photon = $h\nu$, and that of scattered photon = $h\nu'$.

So decrease in energy of photon = $h\nu - h\nu'$. Let mc^2 be the energy of the recoil electron and m_0c^2 be the rest mass energy of the electron.

$\therefore$ K.E. of recoil electron = $(m - m_0)c^2$

which corresponds to the decrease in the energy of the incident photon

∴ K.E. of the recoil electron

$$E = h(\nu - \nu') = h\nu\left(1 - \frac{\nu'}{\nu}\right) = h\nu\left(1 - \frac{c\lambda}{c\lambda'}\right)$$

$$= h\nu\left(1 - \frac{\lambda}{\lambda'}\right) = h\nu\left(\frac{\lambda' - \lambda}{\lambda'}\right) = h\nu\frac{\Delta\lambda}{\lambda + \Delta\lambda}$$

Now $$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

∴ K.E. of recoil electron

$$E = \frac{h\nu \times \frac{h}{m_0 c}(1 - \cos\theta)}{\lambda + \frac{h}{m_0 c}(1 - \cos\theta)}$$

$$= \frac{h^2\nu}{m_0 c\lambda}\frac{(1 - \cos\theta)}{1 + \frac{h}{m_0 c\lambda}(1 - \cos\theta)} \quad \text{[Dividing by } \lambda]$$

$$= \frac{\frac{h^2\nu^2}{m_0 c^2}(1 - \cos\theta)}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos\theta)} \quad [\because\ c = \nu\lambda]$$

which shows that K.E. of the recoil electron depends upon the scattering angle θ.

(*i*) When θ = 0, E = 0

i.e., the electron will not recoil if the scattering photon has scattering angle θ = 0.

(*ii*) When θ = π/2

$$E = \frac{\frac{h^2\nu^2}{m_0 c^2}}{1 + \frac{h\nu}{m_0 c^2}} = \frac{h\nu\alpha}{1 + \alpha}$$

(where $\alpha = h\nu/m_0 c^2$)

(*iii*) When θ = π,

$$E = \frac{\frac{2h^2\nu^2}{m_0 c^2}}{1 + \frac{2h\nu}{m_0 c^2}} = \frac{2h\nu\alpha}{1 + 2\alpha} = \frac{h\nu(2\alpha)}{1 + 2\alpha}$$

So we see that

(*a*) where $\theta = 0$, $E = 0$ *i .e.,* minimum of energy is transferred to recoil electron.

(*b*) where $\theta = \pi$, $E = h\nu \times \dfrac{2\alpha}{1+2\alpha}$ *i.e.,* maximum of energy is transferred to electron.

However $\dfrac{2\alpha}{1+2\alpha} < 1$,

which means that the energy transferred to the electron is less than the energy of incident photon, *i.e., photon cannot transfer whole of energy to the electron in photo-electric effec,t.*

Experimental Set up of Compton Effect

Fig. 6.7 shows the experimental set up for the study of Compton effect. A monochromatic beam of X-rays is allowed to fall on a carbon block B which acts as a scatterer. Normally Compton effect can be best observed in case of elements of low atomic number because the binding energies of the electrons, in these elements are negligible as compared to the quantum energy of the incident photon. The scattering of X-ray photons can take place in different directions and their intensities and wavelengths can be measured by Bragg's spectrometer.

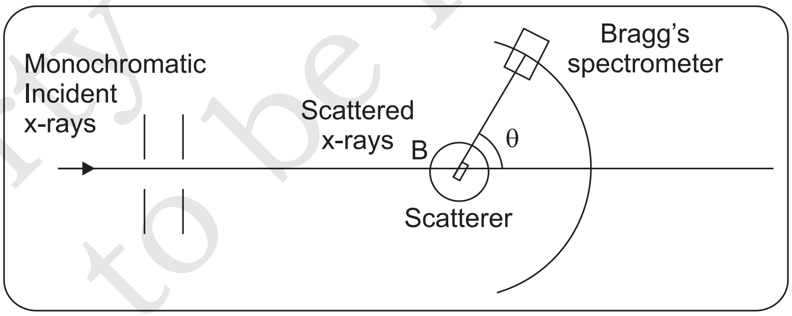


Fig. 6.7

In a typical experiment, K_α line of molybdenum was scattered by gyaphite and the distribution of intensities was studied at different angles. If a graph is plotted between intensities and wavelengths, then we observe that for each value of θ, there are distinct intensity peaks for two wavelengths, one of which corresponds to the incident radiation λ and the other has a higher value λ'. The peak corresponding to λ' is called modified peak.

We can see that with increase of θ, λ′ – λ increases and the shift in wavelength Δλ increases in accordance with the results obtained by Compton and so the Compton effect is verified experimentally.

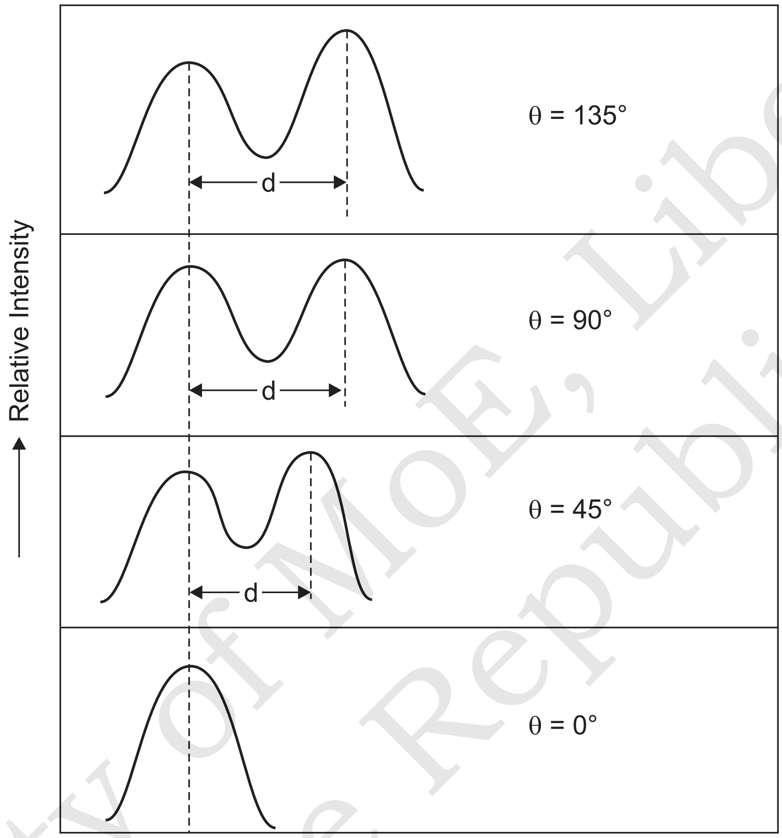


Fig. 6.8

Example 1: *Show that the energy lost by a photon in Compton interaction with a free stationary electron can be written as* $h\nu\left[\dfrac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}\right]$

where $\alpha = \dfrac{h\nu}{m_0c^2}$.

Solution: We have the change in wavelength given by relation

$$\lambda' - \lambda = \frac{h\nu}{m_0 c}(1 - \cos\theta) \qquad \ldots(29)$$

Also, when a photon of energy $h\nu$ is scattered by a free electron, it imparts some energy to the electron and scatters with less energy $h\nu'$. The loss of energy is given by

$$h\nu - h\nu' = h(\nu - \nu') = h\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right)$$

$$= hc\frac{(\lambda' - \lambda)}{\lambda\lambda'} \quad \text{...(30)}$$

Putting the value of λ' from (29) in (30), we get loss of energy as

$$\text{Energy lost} = \frac{hc\left[\dfrac{h}{m_0 c}(1 - \cos\theta)\right]}{\lambda\left[\lambda + \dfrac{h}{m_0 c}(1 - \cos\theta)\right]}$$

$$= \frac{h^2}{m_0}\left[\frac{(1 - \cos\theta)}{\lambda^2\left[1 + \dfrac{h}{m_0 c\lambda}(1 - \cos\theta)\right]}\right]$$

$$= \frac{h^2\nu^2}{m_0 c^2}\left[\frac{(1 - \cos\theta)}{\left[1 + \dfrac{h}{m_0 c\lambda}(1 - \cos\theta)\right]}\right]$$

$$= h\nu \times \frac{h\nu}{m_0 c^2} \times \left[\frac{1 - \cos\theta}{1 + \dfrac{h}{m_0 c^2}(1 - \cos\theta)}\right]$$

$$= h\nu\left[\frac{\alpha(1 + \cos\theta)}{1 + \alpha(1 - \cos\theta)}\right]$$

Example 2: *X-ray with λ = 1 Å are scattered from a carbon block. The scattered radiation is viewed at 90° to the incident beam. (a) What is the Compton shift $\Delta\lambda$? What K.E. is imparted to the recoiling electron?*

Solution: (*a*) We have the Compton shift given by

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

In this case, $\theta = 90°$, $\cos 90° = 0$

$$\Delta\lambda = \frac{h}{m_0 c} = \frac{6.62 \times 10^{-34} \text{ Js}}{9.1 \times 10^{-31} \text{ kg} \times 3 \times 10^{8} \text{ ms}^{-1}}$$

$= 2.43 \times 10^{-12}$ m

= 0.0243 Å

(*b*) From the conservation law of energy, we have

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + \text{K}$$

where K is the K.E. of recoil electron,

Also, $\lambda' = \lambda + \Delta\lambda$

$$\therefore \quad \frac{hc}{\lambda} = \frac{hc}{\lambda + \Delta\lambda} + \text{K} \quad \text{or} \quad \text{K} = \frac{hc\Delta\lambda}{\lambda(\lambda + \Delta\lambda)}$$

$$\text{K} = \frac{6.62 \times 13^{-34}\ \text{Js} \times 3 \times 10^{8}\ \text{ms}^{-1} \times 2.43 \times 10^{-12}\ \text{m}}{1 \times 10^{-10}\ \text{m} \times (1 + 0.0243) \times 10^{-10}\ \text{m}}$$

$$= 4.72 \times 10^{-17}\ \text{J}$$

$$= \frac{4.72 \times 10^{-17}}{1.6 \times 10^{-19}}\text{eV} = \mathbf{.295\ eV}$$

Example 3: *Find the wavelength of X-ray photons which produce recoil electrons of energy* 5 *keV in Compton effect. Assume the direction of recoil electron to be in the direction of incident photon while photon is scattered through an angle of 180°.*

Solution: In this case direction of scattered photon $\phi = 180°$

Also, $$\frac{hc}{\lambda} - \frac{hc}{\lambda'} = 5\ \text{keV}$$

$$= 5000 \times 1.6 \times 10^{-19}\ \text{J}$$

$$\therefore \quad \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 8 \times 10^{16}\ \text{J} \qquad ...(31)$$

From conservation law of momentum in the direction of incident photon,

$$\frac{h}{\lambda} + 0 = \frac{h}{\lambda'}\cos 180° + mv \cos 0°$$

$$= \frac{h}{\lambda'}\cos 180° + mv$$

or $$\frac{h}{\lambda} = \frac{h}{\lambda'} \times -1 + mv$$

$$\therefore \quad \frac{h}{\lambda} + \frac{h}{\lambda'} = \sqrt{\frac{1}{2}mv^2 \times 2m}$$

$$= \sqrt{8 \times 10^{16} \times 2 \times 91 \times 10^{-31}} = 38.1 \times 10^{-24}$$

Multiplying both sides by c,

$$\Rightarrow \quad \frac{hc}{\lambda} + \frac{hc}{\lambda'} = 3 \times 10^{8} \times 38.1 \times 10^{-24}$$

$$= 11.448 \times 10^{-17} \text{ J} \qquad \ldots(32)$$

Adding (31) and (32), we get

$$\frac{2hc}{\lambda} = 8 \times 10^{-16} + 11.448 \times 10^{-17}$$

$$= 122.48 \times 10^{-16} \text{ J}$$

or

$$\lambda = \frac{2hc}{122.48 \times 10^{-16} \text{ J}}$$

$$= (2 \times 6.6 \times 10^{-34} \text{ J}) \times \frac{(3 \times 10^{8} \text{ m/s})}{122.48 \times 10^{-16} \text{ J}}$$

$$= 0.324 \times 10^{-10} \text{ m}$$

$$= \mathbf{0.324 \text{ Å}}$$

6.10 SHORTCOMINGS OF OLD QUANTUM THEORY

The quantum theory based on Bohr's quantum condition and Wilson Sommerfield quantization rule for periodic systems could explain only certain limited problems like energy state of hydrogenation, particle in box, harmonic oscillator, rigid rotator etc. Main shortcomings of old quantum theory are:

(*i*) It could not be applied to non-periodic systems.

(*ii*) It could not explain the spectral lines of relatively simple cases like hydrogen molecule and normal helium atom.

(*iii*) It could not explain the relative intensities of spectral lines at all.

(*iv*) It could not explain process connected with the spin of electrons and Pauli's exclusion principle.

(*v*) Bohr's postulate of discrete, non-radiating energy states on which old quantum theory is based, was empirical without having any theoretical ground. The above difficulties have been resolved by the development of a new method of approach–"Wave Mechanics."

In wave mechanics, Schrodinger by using the concept given by de Broglie derived an equation called Schrodinger's equation which is able to explain the complete behaviour of atomic system in many cases.

Further, half integral values and integral values are obtained automatically during the mathematical solution of the equation. Later on, this was refined by Heisenberg, Born, Jordan, Dirac and others, thereby finally leading to development of new quantum theory.

6.11 UNCERTAINTY PRINCIPLE

The reconciliation of the corpuscular nature with the wave character of light (and also of the electron) has been brought about through the modern quantum theory; and perhaps the best known consequence of wave-particle duality is the uncertainty principle of Heisenberg which ‘ can be stated as follows:

If the x-coordinate of the position of a particle is *known to an accuracy* Δx*, then the x-component of the momentum cannot be determined to an accuracy better than* $\Delta p_x \approx h/\Delta x$*, where h is the Planck's constant.*

Alternatively, one can say that if Δx and Δp_x represent the accuracies with which the *x*-coordinate of the position and the *x*-component of the momentum can be determined, then the following inequality must be satisfied

$$\Delta x \Delta p_x \geq h \qquad \text{...(30)}$$

We do not feel the effect of this inequality in our everyday experience because of the smallness of the value of Planck's constant ($\approx 6.6 \times 10^{-27}$ erg-sec). For example, for a tiny particle of mass 10^{-6} g, if the position is determined within an accuracy of about 10^{-6} cm, then according to the uncertainty principle, its velocity cannot be determined within an accuracy better than $\Delta v = 6 \times 10^{-16}$ cm/sec. This value is much smaller than the accuracies with which one can determine the velocity of the particle. For a particle of a greater mass, Δv will be even smaller. Indeed, had the value of Planck's constant been much larger, the world would have been totally different.

REVIEW EXERCISE

A. MULTIPLE CHOICE QUESTIONS (MCQs)

1. Choose the wrong statement about the spin of an electron, according to quantum mechanics:

(a) It is related to intrinsic angular momentum.

(b) Spin is the rotation of an electron about its own axis.

(c) Value of the spin quantum number must not be 1.

(d) +1/2 value of spin quantum number represents up spin.

2. The Quantum Mechanical Model of the atom was proposed by:

(a) Louis de Broglie (b) Erwin Schrodinger

(c) Neil Bohr (d) Werner Heisenberg

3. The wavelength of the matter waves is independent of:

(a) mass (b) velocity (c) charge (d) momentum

4. Assuming the velocity to be the same, which particle is having longest wavelength

(a) an electron (b) a proton (c) a neutron (d) an α-particle

5. The uncertainty principle states that the error in measurement is due to:

(a) dual nature of particles

(b) due to the small size of particles

(c) due to large size of particles

(d) due to error in measuring instrument

6. The Eigen value of a particle in a box is ___________

(a) $\frac{L}{2}$ (b) $\frac{2}{L}$ (c) $\sqrt{\frac{L}{2}}$ (d) $\sqrt{\frac{2}{L}}$

7. Particle in a box can never be at rest.

(a) True (b) False

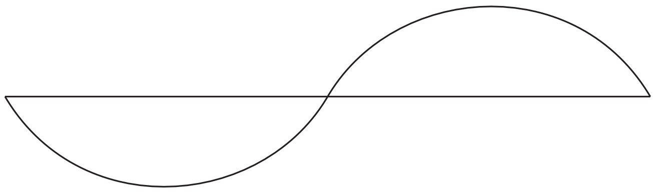
8. What is the minimum Energy possessed by the particle in a box?

(a) Zero (b) $\frac{\pi^2\hbar^2}{2mL^2}$ (c) $\frac{\pi^2\hbar^2}{2mL}$ (d) $\frac{\pi^2\hbar}{2mL}$

9. The wave function of a particle in a box is given by ____________

(a) $\sqrt{\frac{2}{L}}\sin\frac{nx}{L}$ (b) $\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$ (c) $\sqrt{\frac{2}{L}}\sin\frac{x}{L}$ (d) $\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}$

10. The wave function for which quantum state is shown in the figure?



(a) 1 (b) 2 (c) 3 (d)

B. FILL IN THE BLANKS

1. The walls of a particle in a box are supposed to be ___________
2. The wave function of the particle lies in which region ___________?
3. The particle loses energy when it collides with the wall ___________.
4. The Energy of the particle is proportional to _________
5. For a particle inside a box, the potential is maximum at x = __________

C. VERY SHORT ANSWER TYPE QUESTIONS

1. If the uncertainty in the velocity of a moving object is 1.0×10^{-6} ms^{-1} and the uncertainty in its positions is 58 m, the mass of this object is approximately equal to that of:
2. Define the law of Stefan-Boltzmann.
3. Mention the uses of an electron microscope.
4. Explain Planck's hypothesis or what are the postulates of Planck's quantum theory?
5. What is the wavelength associated with a photon of a light with the energy is 3.6×10^{-19} J?

D. SHORT ANSWER TYPE QUESTIONS

1. Describe Planck's three experimental observations that explain the photoelectric effect.
2. What is the difference between classical and quantum mechanics? Provide the equation relating the energy of emitted radiation to frequency.
3. State Heisenberg's uncertainty principle. Give its mathematical expression.
4. An electron and a photon each have a wavelength of 1.00 nm. Find
   (i) their momentum,
   (ii) the energy of the photon, and
   (iii) the kinetic energy of the electron.

5. Find the uncertainty in the position of an electron when the mass of an electron is 9.1×10^{-28} g and the uncertainty in velocity is equal to 2×10^{-3} cm/sec.

E. LONG ANSWER TYPE QUESTIONS

1. A photon of energy 1.02 MeV is scattered through 90° by a free electron. Calculate the energy ot photon and electron after interaction.

2. When ultraviolet radiation of wavelength 1200 Å. is incident on a photo sensitive surface, the electrons emitted have a stopping potential of 5.6 volt. Calculate the work function, threshold frequency and cut off wavelength.

3. On increasing the wavelength of the incident radiation from λ_1 to λ_2 the stopping potential of the photoelectrons emitted is changed by V. Calculate *h*, Planck's constant.

4. Electrons are emitted with zero velocity from a certain metal surface when it is exposed to radiation of wavelength λ = 6800 Å. Calculate threshold frequency and work function of metal.

5. A photoelectric surface has a work function of 4 eV. What is the maximum velocity of photoelectrons emitted by light of frequency 10^{15} Hertz incident on the surface.